ACADEMIC STUDIES IN MATHEMATIC AND NATURAL SCIENCES-2019

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Editor
Prof. Dr. Rıdvan KARAPINAR

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PREFACE

‘Academic Studies in Mathematic and Natural Sciences is serving an academic forum for both academics and researchers working in such fields. mathmematic and natural sciences research is an interdisciplinary by nature. So it covers several fields Besides, have been used as a research method for the contemporary issues relevant to mathmematic and natural sciences.In this book, the academics working in different fields share their results with the scientific community. Thus more researchers will be aware of these studies and have some new ideas for their future studies. The selected articles have been reviewed and approved for publication by referees. It is hoped that the book will be of interest and of value to academics and researchers.

Prof. Dr. Rıdvan KARAPINAR

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COMPARISON OF DIFFUSION COEFFICIENTS WITH DIFFERENT TRANSITION METAL COATINGS IN MGB_2 SUPERCONDUCTOR

Asaf Tolga ÜLGEN*

Introduction

Discovery of superconductivity raised prospects for applications in industrial, energy, magnet technology and triggered abundantly take care of application. The applications of superconducting technology range from the design of industrial electrical-power devices (such as transformers, magnetic energy storage systems, electrical transmission cables, fault current limiters) to the first superconducting wind power generator and magnetic resonance imaging (MRI) imaging, now. The industrial applications of these low critical temperature superconductors were restricted by the high cost of the liquid helium and the complicated operation of the cooling system. Each superconducting material has many advantages and disadvantages in terms of industrial applications. One of these advantages is the boron (B) based superconductor, which is the lowest cost of magnesium diboride (MgB_2). In addition, this cost is expected to decrease significantly as the amount of MgB_2 produced increases, because other superconductors originate from cost-used materials, while they are caused by investment costs in MgB_2. Another advantage of MgB_2 with its low cost is its mechanical properties, lightness and usability in the 10-30 K temperature range (Sander et al., 2013).

The transition metals with high electrical conductivity (such as Fe, Ni and Au) (Gasparrini, 2012) are highly preferred in the production of superconducting wire and tape. The increase in the hardness ratio of the sheath material in the mono and multi-filament wires prepared using the sheath materials such as Cu, Fe, Ni and monel affects the MgB_2 core density (Akdogan et al., 2015). It has been shown in many studies that it has increased the self-field critical current density (J_c) in relation to these studies. In addition, many research groups and studies by our laboratory team show that; it is possible to increase the self-density of MgB_2 wires/tapes with porous structure with the help of external factors and improve the transport properties of the wires. Especially, the development of B-based MgB_2 superconducting technologies that can compete with conventional superconductors when high magnetic field is applied is a current research field. Furthermore, the production of nano-carbon reinforced MgB_2 wire in industrial application has a non-homogeneous core. In studies conducted on carbon-doped MgB_2 wires, it was found that

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the increase in J_c when the high magnetic field was applied could be achieved in nano-SiC (Silicon carbide) doped wires.

Phase transformations of solid, liquid and gas materials stabilization of internal structures and the realization of heat treatments are based on diffusion and are controlled by diffusion. The diffusion is an atomic transfer in solids due to temperature, time, pressure, and substance density, but can also be expressed as spontaneous, spread-dispersion or matter movement generated by the random movement of molecules (Guner et al., 2019). Statistically diffusion (Kennedy and O'Brien, 1972) is that atoms and/or molecules immigrate to a region where their concentration is lower than in a region of high concentration. In the formation of a homogenous crystal structure (microstructure), it is desirable to accelerate the diffusion of the materials (Fe, Ni and Au) which are consciously selected to produce a grain size and preferential crystal orientation (especially to the surface area of the materials). However, it is also possible to improve the properties of the material by preventing partial or total diffusion during a definite time.

The diffusion of atoms and/or molecules is a kinetic event and is mathematically expressed by Fick's laws (Fick, 1855). Diffusion occurs in relation to the resistance of molecules (Flynn et al., 1974) and the flux to the diffusion rate from one region to another region of molecules is called diffusion flux, as namely diffusion coefficients. With this coefficient, mathematical analysis of concentrations and material ratios for a system of diffusion can be calculated. For these calculations, Fick's 2nd law is used and for the finite thin film the diffusion coefficient calculation is given as following equation (Abdullaev and Dzhafarov, 1987);

$$N(x,t) = N_0\left[1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right] \quad (1)$$

where $\mathrm{erf}(x/2(Dt)^{1/2})$ is the error function and $y = x/2(Dt)^{1/2}$ is solved by following equation;

$$\mathrm{erf}(y) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^y \exp(-y^2) \quad (2)$$

The $N_0 = N(0,t)$ is the concentration constant over the calculated sample (MgB_2 material) surface. In the equation (1), $N(x,t)$ is the concentration of impurities, where x represents the distance from the surface, the diffusion coefficient is D and the sintering time to provide diffusion t. Effective parameters in diffusion; i) diffusion mechanism, ii) Diffusion temperature, iii) crystal lattice structure of emitting atom (ADF), iv) activation energy, v) concentration of the active substance, vi) density of crystal defects and vii) the presence of atoms other than the atomic atom emitted in the main phase.

Diffusion processes in the improvement and development of superconducting materials has a vital importance place (Guner et al., 2012). The diffusion is used in processing of superconductors in two requisite ways (Goretta and Chen, 2000); 1- mass transport during consolidation by sintering processes, 2- diffusion between superconductor materials and sheaths or coatings.

Diffusion is much faster through a liquid phase than a solid phase. The concentration of intragranular porosity is generally smallest for the samples that are sintered at the lower temperature, these samples also exhibit the slowest rates of grain growth (Goretta et al., 1992; Ozturk et al, 2012; Ulgen, 2019).

If we look at a comprehensive literature review; Saritekin et al. (2014) have examined physical properties of Ag diffused MgB_2 superconductors such as mechanical, electrical and superconducting features. They have obtained D and activation energy for silver (Ag) diffusion in MgB_2 systems by resistivity measurements. They have reported that the activation energy is ~1.05eV. Dogruer et al. (2013) have investigated the influence of Cu diffusion on microstructural and mechanical properties of bulk MgB_2 superconductors and determined the diffusion coefficient and activation energy of Cu diffusion in MgB_2 materials. In their paper, the magnitude of the diffusion has been reported to change between 2.84×10^{-8} and 3.22×10^{-7} cm^2s^{-1} with the activation energy of nearly 1.09 eV. According to Terzioglu et al. (2013), microstructural, superconducting and mechanical properties of the Cu diffused bulk MgB_2 superconductors depend strongly on the sintering time, and they have reported the maximum diffusion coefficient to be about $2.58x10^{-7}$ cm^2/s. Samuely et al. (2007) studied electron doping and scattering effects via aluminium and carbon substitution in MgB_2. They have proved that, the decline in diffusion coefficient at the π indicates in the carbon doping, while the Al substitution leaves the sample in the clean limit. Also, an increase in the ratio of the diffusion coefficient D_σ /D_π leads shift according to the dirty limit theory. Ulgen and Belenli (2017) have studied the MgB_2 fabrication by conventional solid-state reaction technique. They determined the diffusion coefficient using Fick's law and XRD data. The coefficient diffusion of Fe into MgB_2 matrix was calculated as $2.16x10^{-4}\exp(0.983eV/k_BT)$. The highest T_c value (38.23 K) of MgB_2 bulk material was achieved at 800°C/1h. DeFouw et al. (2008), assays on the kinetics of MgB_2 synthesis in situ by synchrotron X-ray diffraction, using pressed compacts of $200-400\mu m$ magnesium powders mixed with three types of sub micrometres, amorphous, high-purity boron powders. The reaction times can be defined by the diffusion-controlled models of a reacting sphere with kinetics characterized by diffusion coefficients increasing with temperature from $2x10^{-13}$ to $3x10^{-12}$ cm^2s^{-1}, with activation energies of

1.2−1.5 eV. To summarize, the diffusion process in industrial applications finds a significant use in surface processing technology in the ever-growing manufacturing industry. The surface treatments applied in the manufacturing industry include the production of very hard surface layers which are resistant to corrosion and corrosion. The production of such surface layers is based on the principle of diffusion through the penetration of atoms such as carbon, nitrogen and boron into the surface region and the formation of suitable inter-metallic phases. However, the light weight of the MgB_2 superconducting material with a bulk density of 2.6 g/cm^3 is materials with the highest cost and the most suitable for practical applications with its low cost, mechanical properties and critical superconductivity values suitable for technological applications (Cooley et al., 2005).

Sources of the mineral B, which most in Turkey for converting high value-added products MgB_2 superconducting bulk materials were produced. The experimental studies carried out within the scope of the international book chapter have been completed in a way to include compliance. To obtain MgB_2 superconductor, we have used amorphous B in domestic production (90-95% purity produced by Pavezyum Inc.). It was determined that MgB_2 superconducting material produced by conventional solid-state reaction method and it affects the structural, electrical and magnetic properties of this conductor.

We have compared the production parameters by examining the properties of Fe, Ni and Au thin-film coated MgB_2 bulk samples. The diffusion rate and atom/ion migration directions for each metal were also determined. By calculating the diffusion coefficient, the activation energy values required for the interaction were also calculated. These three transition metals have proven to be compatible with the diffusion mechanism in MgB_2 samples. Experimental data were presented to the literature on the selection of suitable sheaths for MgB_2 superconducting wires/tapes. With this study, we think that we have largely eliminated the lack of literature.

Materials and Methods

The starting superconducting powders are commercially available powders of magnesium (Pavezyum company atomized magnesium, purity of 99%, particle sizes ~100 μm), amorphous boron (Pavezyum company boron of purity 95–97%, particle sizes <1 μm), and amorphous nano-boron (Pavezyum company nano-boron, purity of 98.5%, particle sizes <250 nm) from Pavezyum Advanced Chemicals from the Turkey and Alfa Esaer. The ex-situ MgB_2 powder mixture was not used as received, and the amount to be used from this powder (14-16 *g*) was measured using the agate mortar under argon (Ar) gas in the glove box. Throughout the mixing, the agate

ball-powder mass ratio was determined to be 4:1. After mixing the samples with ball milling, we made manual mixing with an agate mortar in glow box an Ar gas to obtain a completely homogeneous mixture. The mixture powder is then pressed into rectangular bars of 25x4x2 mm^3 by using a hydraulic press (Tümas hydraulic) with an applied load of 400 MPa at room temperature. The fabrication process is shown schematically in Figure 1.

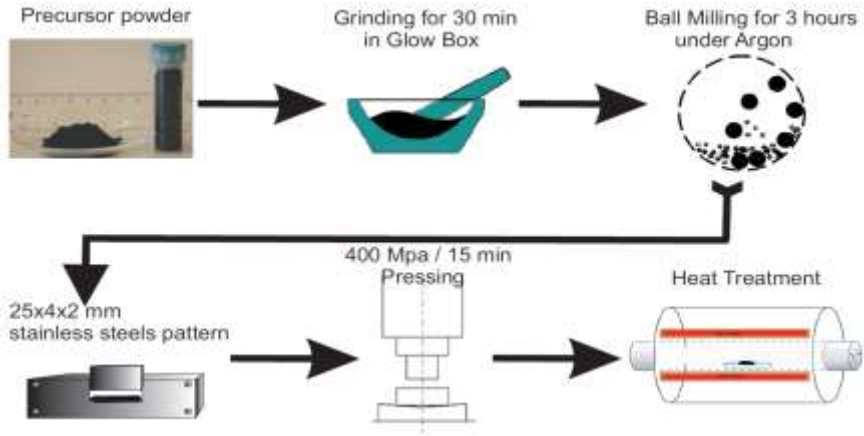

Figure 1: Schematic illustration of fabrication process for the MgB_2 bulk samples.

The MgB_2 bulk samples were annealed for 1 hour at different five temperatures (650, 700, 750, 800 and 850ºC) and heat treatments were performed under Ar gas at 5-10 bar pressure using a horizontal tube furnace (Protherm-Model PTF12/75/200). For annealing process of the bulk samples, a stainless-steel tube with vacuum and Ar gas inlets was used. The heating and cooling rates are adjusted to be 5ºC/min. The temperature profile along this furnace was determined with a thermocouple and calibrated.

Different transition metal coatings in MgB_2 superconductor is applied on bulk pellets by evaporation of Fe, Ni and Au metal films using EDWARDS Auto 306 Vacuum Coating Systems under $1.1x10^{-4}$ Pa vacuum. The Auto 306 is designed for physical vapour deposition processes under high vacuum. Resistance evaporation is often simplest, and most effective deposition technique. In this technique, commonly used metals such as iron, chromium, silver, gold and many others are readily evaporated. For the calculation of the diffusion rate, diffusion mechanism and activation energies of these three highly conductive metals on coated MgB_2 bulk samples, removal of surface layer was achieved by rubbing against an unused fine grinding paper (P2000 and P3000) under 1.5 N weight, as can be seen in the Figure 2.

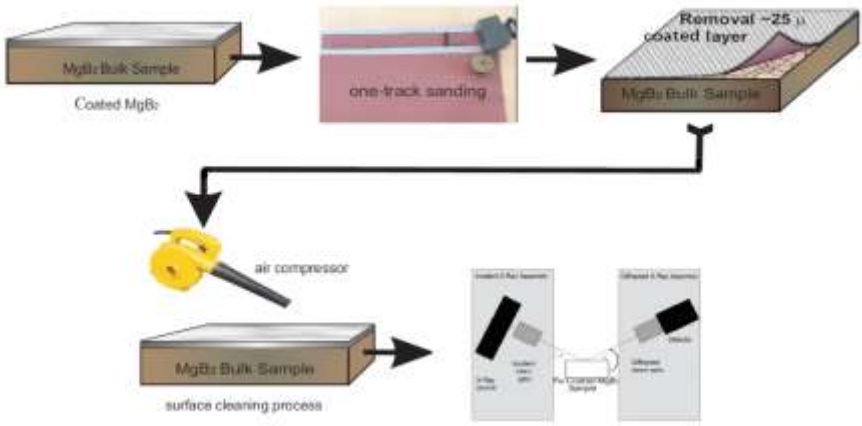

Figure 2: The removal process in MgB_2 superconducting material.

By applying constant weight and rubbing on an unused grinding paper just once for constant length, we achieved almost equal removals of 25 μm layer from the coated surface. After each removal process was completed from the carefully cleaned surface using a high amount of air blower, X-Ray powder diffraction patterns of the coated-sintered ex-situ MgB_2 samples were obtained with a Rigaku Multiflex+XRD in the range $2\theta = 10 - 90^o$ at a scan speed of 5^omin^{-1} and step increment of 0.02^o at room temperature. All removals layers from the metal (Fe, Ni and Au) coated surface of ex-situ superconducting samples were done in eight steps corresponding to a depth of 195-205 μm. We record the experimental resistivity-temperature (ρ-T) curves of 20 - 45 K with the aid of the four-point contact method by using Helium closed-cycle cryostat system (CRYO Industries) with provision for the vacuum. In the normal state above critical temperature (T_c), all MgB_2 superconductor samples showed metallic behaviour.

Results and Discussion

Figure 3 shows the X-ray diffraction patterns of Fe, Ni and Au coated MgB_2 samples fabricated sintering process at 800°C for 1 hours. Result of XRD of the following examples which are noncoated (bare) MgB_2 (black line), Ni coated MgB_2 (blue line), Fe coated MgB_2 (red line), and Au coated MgB_2 (green line) that can be seen in Figure 3. It can be said that main phase (101) of the MgB_2 matrix can be obtained from the X ray diffraction analyses of all samples.

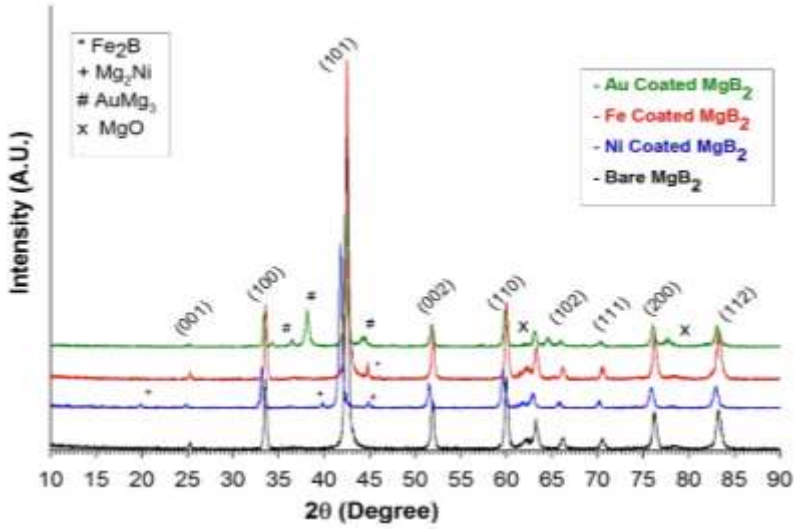

Figure 3: XRD patterns of MgB_2 samples bare and after metal coating. From bottom to top; as received bare sample, Ni coated, Fe coated and Au coated MgB_2 pellets sintered at 800°C/1h.

XRD pattern of Ni, Fe (Ulgen and Belenli, 2017) and Au (Ulgen, 2017) coated MgB_2 sample taken before noncoated has an only peak of this superconductor, while after coated Ni, Fe and Au coated MgB_2 sample have Mg_2Ni, Fe_2B and $AuMg_3$, respectively. On the other hand, the characteristic MgB_2 superconducting and MgO impurity phases appear at the crystal structure. In addition, it is found that the intensity of the XRD peaks belonging to MgO phase increases after sintering process. Table 1 shows the lattice parameters *c* value of Ni, Fe (Ulgen and Belenli, 2017) and Au coated (Ulgen, 2017) and bare MgB_2 pellets for different sintering temperatures. The lattice parameters *c* of the coated samples is found to become larger with increasing sintering temperature as seen in Table 1. However, it is very difficult to understand the increase in the *c* parameter. Metal (Ni, Fe and Au) ions within MgB_2 lattice structure may both be disturbing (distorting), because of its ionic radius is smaller than the atomic radius.

Table 1: Lattice parameter *c* (Å) values of different annealing temperatures for bare and Ni, Fe and Au coated MgB_2 pellet samples.

Sintering Conditions	**Bare MgB_2**	**Ni Coated MgB_2**	**Fe Coated MgB_2**	**Au Coated MgB_2**
650°C/1h	3.524	3.532	3.531	3.552
700°C/1h	3.521	3.536	3.529	3.543

750ºC/1h	3.522	3.539	3.530	3.542
800ºC/1h	3.529	3.541	3.526	3.539
850ºC/1h	3.528	3.543	3.540	3.532

The surface morphology of the metal coated and bare samples of MgB_2 were observed by scanning electron microscope (SEM) are displayed in Figure 4. These findings may be due to prevention of Mg loss from the outer surface during annealing because of metal layer.

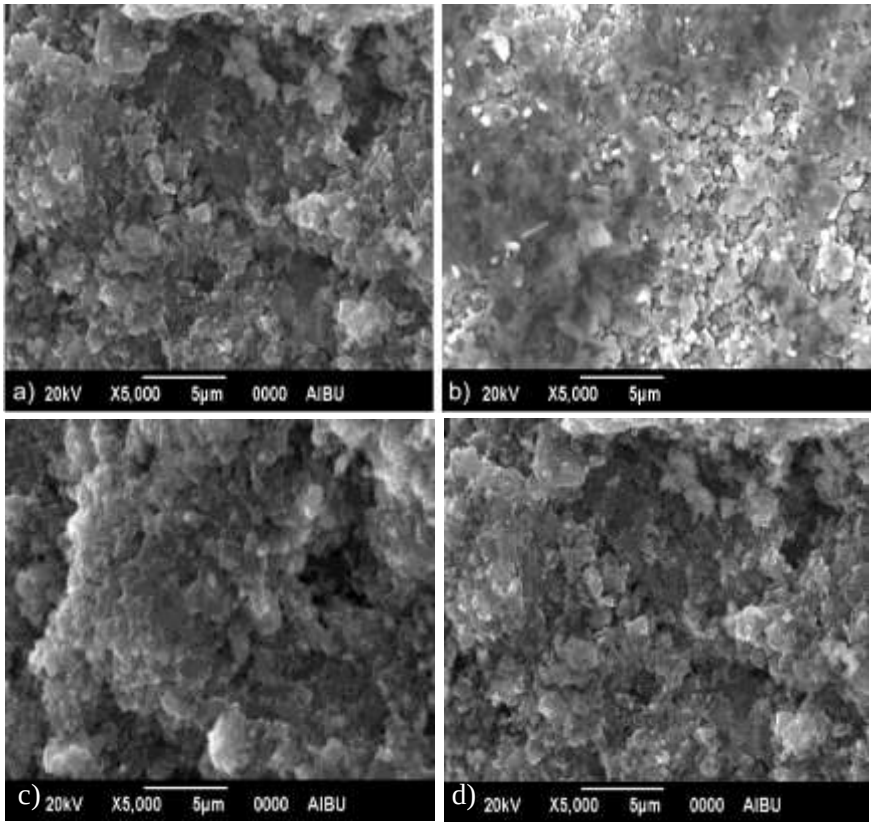

Figure 4: SEM images taken from coated surface for a) Bare, b) Fe coated, c) Ni coated and d) Au coated MgB_2 pellet samples sintered at 800°C for 1 hours.

Also, the grain structure and morphology seem to be different in these metal (Fe, Ni and Au) coated samples where the grains have inosculated into big clusters. Therefore, being founded on the SEM observations, it seems that the different transition metal coated samples are denser and have better connected grains than the bare ones.

Table 2 gives the diffusion coefficients calculated by using equation (1) and the change in depth of the c-parameter values of metal-coated ex-situ MgB_2 samples. We are functioning depth profile for the changes in these physical quantities by successive removal of thin layers (about of 25 μm) from metal diffused surface of the MgB_2 samples. The data on the concentration profile of the diffuse impurity in the metal diffused sample can be determined by measuring the distribution of the deformation ($\Delta c/c_0$) on the thickness of a sample. Where Δc is the change in the value of lattice parameter c with respect to each removal process and c_0 is lattice parameter $_c$ value of bare MgB_2 superconductor sample. In additional, the diffusion coefficient versus 1/T is used to determine the activation energy required for the diffusion of metals into MgB_2 pellet and represents the ratio of the D diffusion process in the Arrhenius curve.

Table 2: Diffusion coefficient (cm^2/s) of different annealing temperatures for Ni, Fe and Au coated MgB_2 pellet samples.

Sintering Conditions	**Ni diffusion coefficients**	**Fe diffusion coefficients**	**Au diffusion coefficients**
650ºC/1h	$4.1578x10^{-10}$	$8.8660x10^{-10}$	$2.5864x10^{-10}$
700ºC/1h	$9.2829x10^{-10}$	$3.0789x10^{-9}$	$6.0149x10^{-9}$
750ºC/1h	$1.9431x10^{-9}$	$5.3192x10^{-9}$	$3.4481x10^{-9}$
800ºC/1h	$4.0274x10^{-9}$	$7.2495x10^{-9}$	$7.7517x10^{-9}$
850ºC/1h	$7.1186x10^{-9}$	$9.7812x10^{-9}$	$8.9865x10^{-9}$

It is found that the metal diffusion coefficient decreases with increasing sintered temperature, as expected. Besides, the diffusivity between the materials coated with Ni, Fe and Ni is $2.6785x10^{-9}$, $2.16x10^{-4}$ and $1.5392x10^{-9}$, respectively. At the same time, this relationship suggests that the metals are also associated with ionic radius. As known in the literature, the ionic radius of Fe, Ni and Au metals are 0.645 Å, 0.690 Å and 0.850 Å, respectively. According to both low temperature-dependent D_{Ni}, D_{Fe}, D_{Au} and also low Q values inferred to the Arrhenius equation in Figure 5, the required minimum Q for the entry of transition metal atoms/ions to the MgB_2 crystal structure is about 1.013 eV, 0.983 eV and 1.35 eV, respectively.

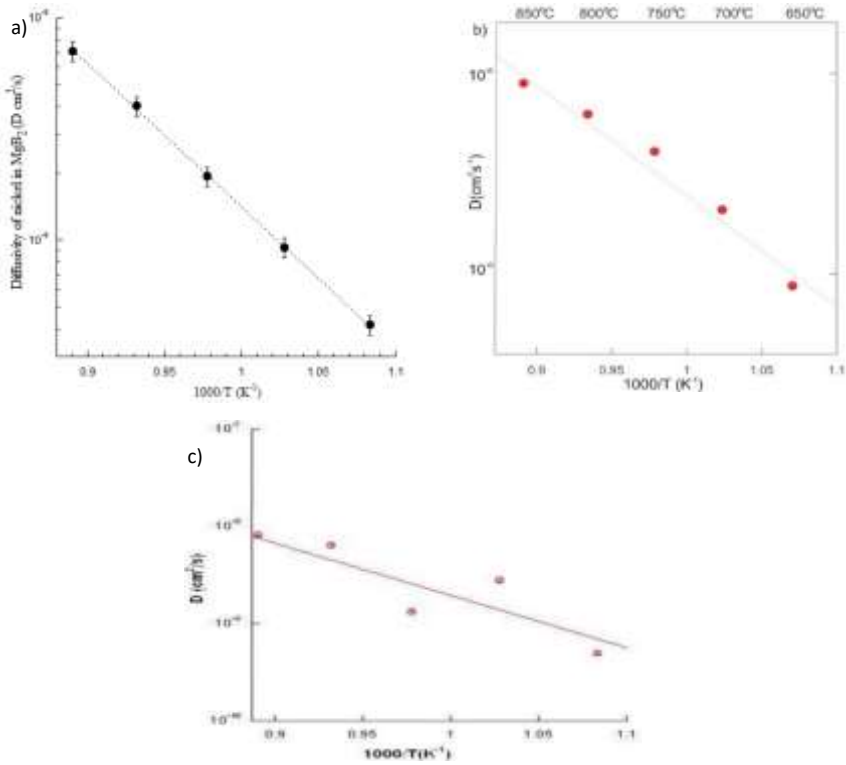

Figure 5: Temperature-dependent a) Ni b) Fe and c) Au diffusion rate at the temperatures between 650°C and 850°C for the bulk MgB_2 superconducting material.

Conclusions

One surface of each bulk material was coated approximately 50 μm with the iron (Fe), nickel (Ni) and gold (Au) different transition metals having high electrical conductivity by using metal evaporation system. So, the interface interactions of the superconducting bulk materials as MgB_2-Fe, MgB_2-Ni and MgB_2-Au obtained from ex-situ bulk samples coated with transition metals were investigated in detailed. Diffusion coefficient, diffusion rate and activation energy values of ex-situ superconducting MgB_2 bulk samples were obtained for each transition metals. In addition, micro and macro structural, superconducting properties of MgB_2 bulk materials coated by using three different metals were determined. As a conclusion, MgB_2 superconducting samples with high and convenient performances can be produced by sintering at nominal temperatures for extended duration time.

Acknowledgements

This work is supported by the Scientific and Technological Research Council of Turkey, (Project no:117F263) and in part by Sirnak University Research Fund Grant No:2017.03.02.01.

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RADYASYON ÖLÇÜMLERİNDE KULLANILAN YENİ TİP CİHAZLAR: SANAL CİHAZLAR

New Type Instruments used in Radiation Measurements: Virtual Instruments

Gözde TEKTAŞ*

1. Giriş

Bilimsel hesaplama (scientific computing) terimi bilim ve mühendislik ile ilgili problemleri çözme amacıyla bilgisayarların kullanımını tanımlamak için yıllardan beri kullanılmaktadır. Bilimsel hesaplama uygulamaları verilerin elde edilmesi (data acquisition), verilerin analiz edilmesi (data analysis) ve verilerin görüntülenmesi (data visualization) olmak üzere üç aşamalı işlemden oluşmaktadır (Jerome, 2010).

Sanal cihazlar modern laboratuvarların temelini oluştururlar. Bir sanal cihaz; bilgisayar, yazılım ve donanımdan oluşmaktadır. Bunlar gerçek cihazların işlevlerini yerine getirmek için bir araya getirilirler (Travis ve Kring, 2006). Donanım, PC'ye bir arayüz yolu ve harici girişlerin (external inputs) bağlanmasını sağlar. Yazılım, cihazın kontrol edilmesini, verilerin toplanmasını, işlenmesini, görüntülenmesini, analiz edilmesini ve kaydedilmesini sağlar (Tooley, 2005). Sanal cihaz tasarımı ile mühendisler ve bilim adamları kısa sürede yüksek kaliteli ve düşük maliyetli ürünler tasarlayabilirler (Jerome, 2010).

Gerçek cihazlar satıcının belirlediği işlevlere sahipken, sanal cihazların işlevleri kullanıcı tarafından tanımlanır. Gerçek cihazlarda bulunan özel bileşenler ve devre dışında, bağımsız cihazların genel yapısı PC tabanlı sanal cihazlarınkine çok benzerdir. Bir sanal cihazı gerçek bir cihazdan farklı yapan şey, kullanıcının özel ihtiyaçlarına göre değiştirilebilmeleridir. Gerçek bir cihaz, özel bir takım veri işleme fonksiyonlarını gerçekleştirmek için entegre bir devre içerir. Sanal bir cihazda, bu fonksiyonlar PC işlemcisi üzerinde çalışan yazılım ile gerçekleştirilir (Jerome, 2010).

Geleneksel programlama dil ortamı ve grafiksel programlama ortamı sanal cihaz geliştirme ortamlarıdır. 1980'lerin ortalarında ve sonlarında, yeni programlamlama dilleri yüksek seviyeli programlamaya izin verdikleri için (örneğin C dili) yaygınlaşmaya başlamışlardır. Yeni nesil

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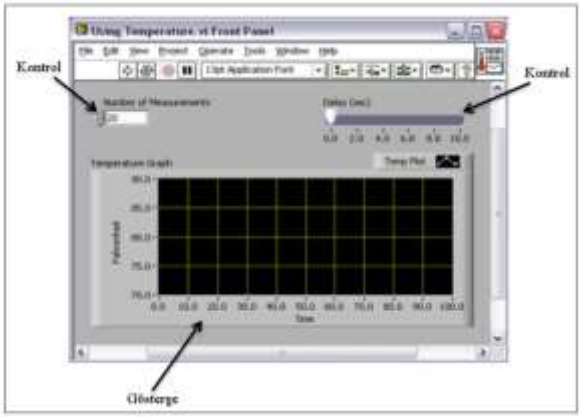
grafiksel programlama araçları kullanıcılar için sistem entegrasyonuna olanak sağlar. Bu programlama araçlarından biri de LabVIEW yazılımıdır (https://obren.info/papers/virtualinstrumentation.pdf, 2019).

LabVIEW (Laboratory Virtual Instrument Engineering Workbench), araştırma laboratuvarları, akademi ve endüstri genelinde yaygınlaşan bir grafiksel programlama ortamıdır. LabVIEW kullanılarak ihtiyaç duyulan sanal cihaz oluşturulabilir (Jerome, 2010).

LabVIEW program geliştirme ortamı standart C ve Java geliştirme sistemlerinden farklıdır. Diğer programlamlama sistemleri kod satırlarını oluşturmak için metin tabanlı dilleri kullanırken, LabVIEW "G" olarak adlandırılan grafiksel programlama dilini kullanır (Travis ve Kring, 2006). Bu grafiksel özelliği test ve ölçüm, cihaz kontrol, veri toplama ve analizi uygulamaları için onu ideal yapar (Bitter, Mohiuddin ve Nawrocki, 2007).

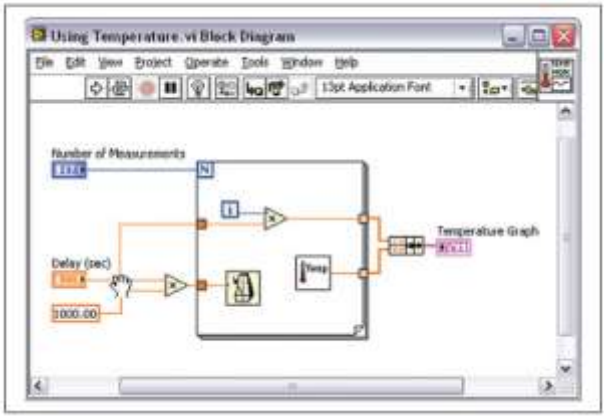
LabVIEW programı ile oluşturulan bir sistem görünüş ve çalışma şekli bakımından laboratuvarlarda kullanılan osiloskop, sinyal jeneratörü gibi cihazlara benzediği için sanal cihaz (virtual instruments, VIs) olarak adlandırılır. LabVIEW ekranı ön panel (front panel) ve blok diyagram (block diagram) olmak üzere iki ana kısımdan oluşur (Ünsaçar ve Eşme, 2009).

Ön panel, tıpkı gerçek cihazların kullanıcı arayüzüne benzer. Sanal cihazın kontrol butonları, düğmeleri ve göstergeler bu kısımda yer almaktadır. Kullanıcı ön paneldeki kontrolleri kullanarak sanal cihazın kontrolünü sağlar. Göstergeler ise elde edilen ya da hesaplanan verileri kullanıcıya sunar. Şekil 1'de örnek bir ön panel ekranı görülmektedir.



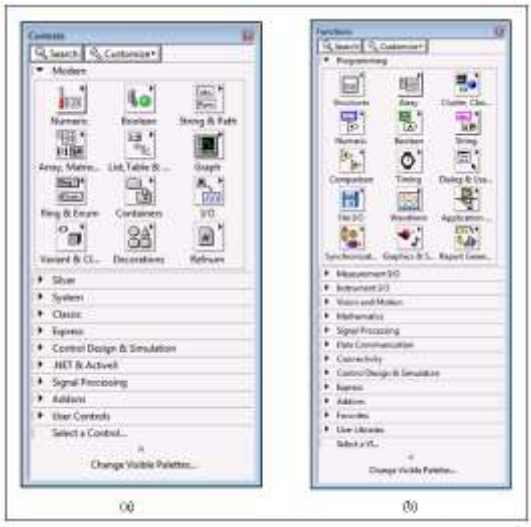
Şekil 1. Ön panel ekranı
(http://www.ni.com/white-paper/7566/en/, 2019).

Sanal cihaz için kod blok diyagram ekranında geliştirilmektedir. Kod, kullanılan kontroller, göstergeler ve fonksiyonlar arasında bağlantılar yapılarak yazılmaktadır. Sanal cihaz burada geliştirilen koda göre çalışmaktadır. Şekil 2'de örnek bir blok diyagram ekranı görülmektedir.



Şekil 2. Blok diyagram ekranı (http://www.ni.com/getting-started/labview-basics/environment, 2019).

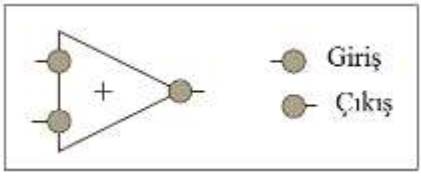
Ön panel ve blok diyagram ekranlarında kullanılmak üzere üç tane palet bulunur. Bunlar kontrol paleti (control palette), fonksiyon paleti (function palette) ve araç paletidir (tools palette). Ön panel ekranında bulunan kontrol paletinde kontroller ve göstergeler bulunmaktadır. Fonksiyon paleti ise blok diyagram ekranında kullanılır. Farklı işlevlere sahip fonksiyonların seçimi bu paletten yapılmaktadır. Kontrol ve fonksiyon paleti Şekil 3'te görülmektedir. Ayrıca araç paleti (tools palette) hem ön panelde hem de blok diyagramda bulunur. Bu palette bulunan araçlar ön panel ve blok diyagramdaki nesneleri işletmek ve düzenlemek için kullanılır (Jerome, 2010). Araç paleti Şekil 4'te yer almaktadır.



Şekil 3. (a) Kontrol paleti ve (b) fonksiyon paleti (http://www.ni.com/getting-started/labview-basics/environment, 2019).

Şekil 4. Araç paleti (http://www.ni.com/getting-started/labview-basics/tools, 2019).

Fonksiyonlar genellikle bir ya da daha fazla giriş kabul eder ve bir sonuç verir. Örneğin, toplama (add) fonksiyonu için kullanılan düğümler (nodes) Şekil 5'te görülmektedir. Toplama fonksiyonu iki giriş (toplanacak değerler) ve bir çıkışa (toplam) sahiptir (Larsen, 2011).

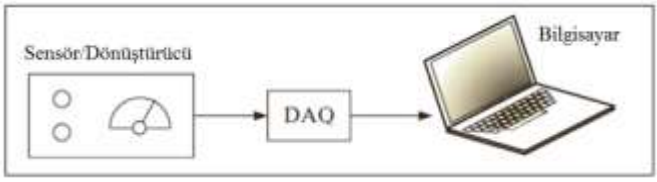


Şekil 5. Toplama fonksiyonun giriş ve çıkış düğümleri (Larsen, 2011).

LabVIEW, sanal cihazları çalıştırmak için veri akış modelini izler. Bir blok diyagram fonksiyonu, girişlerine veri geldiğinde çalışır. Fonksiyon, işlemini tamamladığında çıkış bağlantı ucuna veri gönderilir ve veri akışındaki bir sonraki fonksiyona çıkış verileri iletilir (Jerome, 2010).

LabVIEW, analog ve dijital sinyal üretmek (generate) ve sinyal toplamak (acquire) için veri elde etme (data acquisition, DAQ) cihazlarını kullanır. Ayrıca genel amaçlı arayüz veri yolu (GPIB), ethernet ve seri port üzerinden veri transferini kolaylaştırır (Travis ve Kring, 2006).

Veri elde etmenin amacı, verileri bilgisayarda analiz etmek ve kaydetmek için bir ya da daha fazla laboratuvar cihazından gelen verileri toplamaktır. Basit bir veri elde etme sistemi için bir sinyal çıkışı veren dönüştürücü (transducer), veri elde etme cihazı (DAQ) ve bir bilgisayar gerekir (Şekil 6) (Larsen, 2011).



Şekil 6. Veri elde etme sistemi (Larsen, 2011).

DAQ cihazı, gelen sinyalleri bilgisayarın kullanabileceği dijital sinyallere çevirir. Çok miktarda veri okunmak istendiğinde veri işlerken ve sonuçları görüntülerken yazılım önemli rol oynar. Bu esneklik, bir donanım cihazı ile birçok farklı tipteki sinyalin ölçülmesine olanak sağlar (Ünsaçar ve Eşme, 2009).

Dijital sinyal işleme (digital signal processing), nükleer fizik deneylerinde olduğu gibi tüm mühendislik alanlarında kullanılır. Günümüzde nükleer dijital sinyal işleme sistemleri genellikle LabVIEW grafiksel programlama ortamında oluşturulan sanal cihazlar ile gerçekleştirilir (Folea, 2011).

2. Radyasyon Sinyallerinin İncelenmesinde Kullanılan Cihazlar

Nükleer fizikte radyasyonun varlığını dedekte etmek için çeşitli dedektörler kullanılır. Dedektörler, elektrik sinyalleri formunda dedekte edilen radyasyon hakkında çeşitli bilgiler sağlar. Ancak bu bilgileri elde etmek için alınan sinyaller elektronik sistem tarafından işlenmelidir. Bu sistem çeşitli görevleri gerçekleştirmek için tasarlanabilir. Örneğin, bu sistem dedektörden gelen çeşitli sinyalleri ayırmak için, enerji bilgisini elde etmek için düzenlenebilir (Leo, 1987).

Bu elektronik sistemde sinyallerin işlenmesi, analiz edilmesi ve görüntülenmesi için çeşitli cihazlar kullanılmaktadır. Osiloskop, sayıcı (counter), geciktirici jeneratör (gate and delay generator), koinsidens ünite, ön yükseltici (preamplifier), ana yükseltici (amplifier), tek kanallı analizör (single channel analyzer) ve çok kanallı analizör (multichannel analyzer) bu cihazlara örnek verilebilir. Bu çalışmada osiloskop, geciktirici jeneratör, koinsidens ünite ve sayıcı cihazları ele alınmıştır.

2.1. Osiloskop

Osiloskop, girişine gelen sinyalleri görüntüleyen bir cihazdır. Görüntülenen sinyalin voltaj-zaman değerlerini belirlemek için kullanılabilir. Bir sinyalin maksimum değeri, minimum değeri, genlik ve frekans gibi büyüklükleri osiloskop ile belirlenebilir.

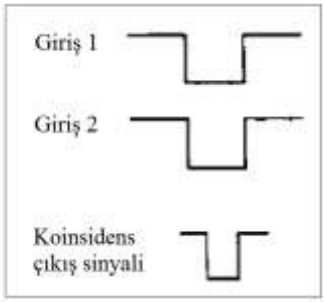
Nükleer fizik deneylerinde osiloskop bir radyasyon dedektörünün oluşturduğu sinyalleri görüntülemek ve bu sinyallerin voltaj - zaman büyüklükleri hakkında bilgi elde edebilmek için kullanılır.

2.2. Geciktirici Jeneratör

Geciktirici jeneratör, girişine gelen pozitif ya da negatif kutuplu mantık sinyallerini zamana göre ayarlanan bir değere geciktirir. Seçilen genlik, polarite ve genişliğe sahip bir çıkış mantık sinyali üretilir (https://groups.nscl.msu.edu/nscl_library/manuals/eggortec/416a.pdf, 2019). Bu cihazlar sadece bir sinyali geciktirmek için değil, aynı zamanda başka bir elektronik üniteyi (örneğin bir sayıcıyı) tetiklemek için de kullanılabilirler (Tsoulfanidis, 1995).

2.3. Koinsidens Ünite

Nükleer ve parçacık fiziğindeki en önemli tekniklerden biri de koinsidens (eş zamanlı) olaylarının belirlenmesidir. Koinsidens ölçümlerindeki temel kaide, sinyal kaynağından (dedektörden) gelen analog sinyalleri mantık sinyallerine dönüştürerek bu sinyalleri koinsidens modüle göndermektir (Leo, 1987). Şekil 7'de görüldüğü gibi, giriş sinyalleri birbirleriyle koinsidens ise bir çıkış sinyali üretilir.



Şekil 7. Koinsidens olayı (Leo, 1987).

Belirli radyasyon türlerine bağlı sinyalleri ayırma işlemi ile sadece belirli bir yönde gelen parçacıklardan ya da tek bir parçacık türünden gelen sinyallerin incelenmesi sağlanabilir. Böyle durumlarda koinsidens ünite kullanılarak istenmeyen sinyallerin ayırılması sağlanabilir (Tsoulfanidis, 1995).

2.4. Sayıcı

Sayıcı, kullanıcı tarafından tanımlanan süre içerisinde girişine gelen sinyalleri sayan bir cihazdır.

Nükleer ve parçacık fiziği deneylerinde bir dedektörden gelen sinyallerin sayısını belirleme temel bir ölçümdür. Örneğin; sayıcı, bir radyasyon kaynağının aktivitesinin belirlenmesinde kullanılan bir cihazdır (Leo, 1987).

3. Radyasyon Sinyallerinin İncelenmesi için Sanal Cihaz Tasarımı

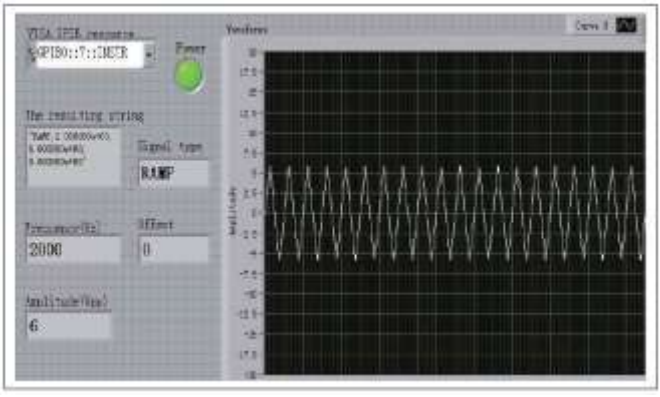
Radyasyon sinyallerinin incelenmesi için kullanılan sanal cihazlar, bilgisayar ortamında LabVIEW yazılım fonksiyonları vasıtasıyla geliştirilebilirler. Geliştirilen sanal cihazlar ile sinyallerin işlenmesi ya da analiz edilmesi sağlanabilir. Burada örnek olarak sanal cihazların bir çeşidi olan sanal osiloskop, sanal sayıcı, sanal geciktirici jeneratör ve sanal koinsidens ünite ele alınmıştır. Bu cihazların donanım kısmında farklı özelliklere sahip sayısallaştırıcılar ya da veri toplama cihazları kullanılabilir. Bu cihazlar ile bir sanal cihaza veri girişi sağlanır. Şekil 8'de USB-5133 model sayısallaştırıcının fotoğrafı örnek olarak verilmiştir.

Şekil 8. Sayısallaştırıcı
(http://www.ni.com/it-it/support/model.usb-5133.html, 2019).

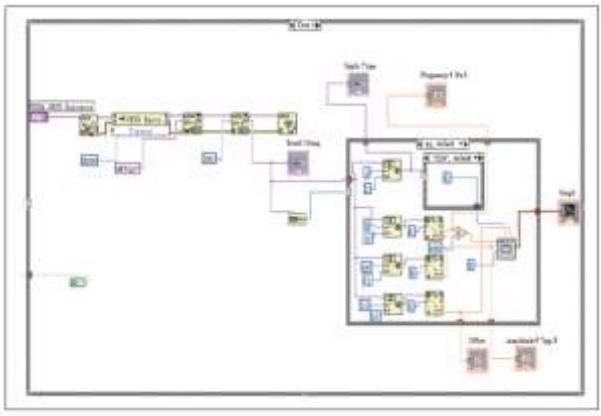
3.1. Sanal Osiloskop

Nükleer fizikte sanal osiloskop, radyasyon ölçümü ve bir puls jeneratöründen elde edilen sinyalleri incelemek için kullanılabilir (Tektaş, 2015). Gerçek osiloskoptaki işlemler sanal osiloskop ile de gerçekleştirilebilir. Gerçek ve sanal osiloskop arasındaki temel fark, sanal osiloskoptaki işlemlerin yazılım fonksiyonları vasıtasıyla gerçekleştirilebilmesidir (Tektas ve Celiktas, 2017).

Sanal osiloskobun yazılım kısmı, ön panel ve blok diagramdan oluşmaktadır. Ön panel kısmı, voltaj ayarı (Volt/division), zaman ayarı (Time/division), yatay konum (horizontal position), düşey konum (vertical position) ve kanal seçimi gibi osiloskop kontrol düğmeleri, ölçüm değerlerini takip edebileceğimiz göstergeler ve bir osiloskop ekranından oluşmaktadır. Veri elde etme, sinyalin görüntülenmesi, voltaj - zaman ölçümleri ve verilerin kaydedilmesi gibi işlemler blok diyagramda gerçekleşmektedir (Tektaş, 2015). Şekil 9 ve 10'da sanal osiloskobun ön panel ve blok diyagram ekranı için bir örnek görülmektedir.

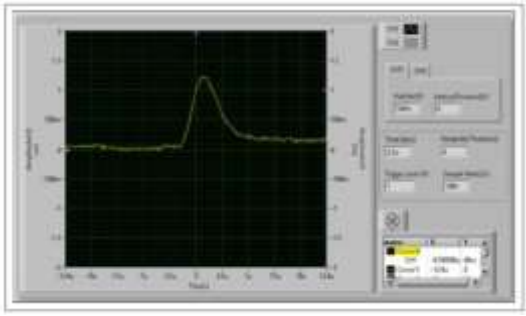


Şekil 9. Sanal osiloskobun ön panel ekranı (Guili ve Quancun, 2013).



Şekil 10. Sanal osiloskobun blok diyagram ekranı (Guili ve Quancun, 2013).

Bir dedektörden ya da fonksiyon jeneratöründen alınan sinyaller sanal osiloskopta görüntülenerek sinyalin voltaj ve zaman ölçümleri yapılabilir. Örneğin sinyalin maksimum voltaj, minimum voltaj, yükselme zamanı (rise time), düşme zamanı (fall time) ve frekans gibi büyüklükleri belirlenebilir. Şekil 11'de tabii fon siyalinin ekran görüntüsü örnek olarak verilmiştir.



Şekil 11. Sanal osiloskopta görüntülenen tabii fon sinyali (Tektas ve Celiktas, 2015).

3.2. Sanal Sayıcı

Sanal cihazların bir çeşidi olan sanal sayıcı LabVIEW yazılımı kullanılarak tasarlanabilir. Üretici firmalar tarafından üretilen gerçek sayıcılar gibi sanal bir sayıcı, belirli bir zaman içerisinde gelen sinyalleri sayar (Tektas ve Celiktas, 2017).

Bir sanal sayıcının ön panel ekranında sayım süresinin kontrol edilebilmesi için bir kontrol düğmesi ve sayım sayısının kullanıcıya

sunulması için de bir gösterge yer alabilir. Blok diyagram kısmında ise gelen sinyallerin sayılması için kod geliştirilmelidir.

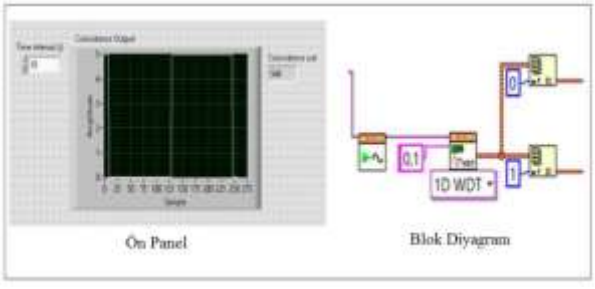
3.3. Sanal Geciktirici Jeneratör

Sanal geciktirici jeneratörün ön panel ekranında gecikme zamanı (delay time), sinyalin genlik (amplitude) ve genişlik (width) değerinin ayarlanabileceği düğmeler yer alabilir. Çıkış sinyalinin kutbunun değiştirilebileceği bir kontrol kullanılabilir. Blok diyagramda ise ön panelde ayarlanan değerlere göre çıkış sinyalinin oluşturulması için işlemler gerçekleştirilmelidir. Örnek olarak Şekil 12'de LabVIEW yazılımı ile geliştirilen sanal geciktirici jeneratörün kullanıcı arayüzü görülmektedir.

Şekil 12. Sanal geciktirici jeneratörün kullanıcı arayüzü (Tektas ve Celiktas, 2018).

3.4. Sanal Koinsidens Ünite

Bilgisayar ortamında geliştirilen sanal cihazların bir çeşididir. Sanal koinsidens ünitenin blok diyagram ekranında her iki giriş sinyali birbiriyle koinsidens olduğunda bir çıkış sinyalinin oluşturulması için kod yazılmalıdır. Şekil 13'te sanal koinsidens ünitenin ön panel ve blok diyagram ekranı için bir örnek görülmektedir.



Şekil 13. Sanal koinsidens ünitenin ön panel ve blok diyagram ekranı (Tektas ve Celiktas, 2018).

4. Sonuç

Sanal cihaz teknolojisinin gelişmesiyle birlikte bu cihazlar birçok alanda kullanılmaya başlanılmıştır. Gerçek cihazlardan farklı olarak, sanal cihazlar bilgisayar ortamında geliştirilir. Yazılım vasıtasıyla tasarlandıkları için kullanıcının uygulama amacına göre çeşitli sanal cihazlar tasarlamak mümkündür.

Günümüzde sanal cihaz tasarlamak için LabVIEW yazılımı kullanılmaktadır. Grafiksel programlama özelliği sayesinde sanal bir cihaz için kolaylıkla kod geliştirilebilir. LabVIEW yazılımının kütüphanesinde farklı işlevlere sahip birçok fonksiyon mevcuttur. Örneğin, bu fonksiyonlar vasıtasıyla sinyallerin görüntülenmesi, işlenmesi ve analiz edilebilmesi mümkündür.

Nükleer fizik deneylerinde bir dedektörden alınan sinyallerin işlenmesi, analiz edilebilmesi ve görüntülenmesi için birçok cihaz kullanılmaktadır. Burada osiloskop, koinsidens ünite, geciktirici jeneratör ve sayıcının işlevlerinden bahsedilmiştir. Bu cihazların LabVIEW yazlımı ile tasarlanabileceği görülmüştür.

Sanal cihazların deneylerdeki kullanım amaçlarına göre tasarlanabilmesi ve gerektiğinde ihtiyaca göre yeniden düzenlenebilmesi bu cihazlara çok büyük bir avantaj sağlamaktadır. Sadece nükleer fizik deneyleri için değil, diğer bilimsel çalışmalar için de LabVIEW ile sanal cihaz tasarımı yapılabilir.

Teşekkür

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YARI DÜZGÜN KONVEKS BİR ÇOKYÜZLÜ İLEn TANIMLI UZAYIN UZAKLIK FORMÜLLERİ

Distance Formulae of A Space Induced By A Semi-Regular Convex Polyhedra

Zeynep CAN[*] & Zeynep ÇOLAK[**]

Giriş

Her bir yüzü düzlemsel çokgenler ile sınırlanan, ayrıt ve köşeleri de bu çokgenlerin kenar ve köşeleri olan cisimlere çokyüzlü denilmektedir. Herhangi iki noktasını birleştiren doğru parçasının tamamı, çokyüzlünün yüzeyinde veya içinde kalıyorsa bu çokyüzlüye dış bükey (konveks) çokyüzlü denilmektedir.

Özellikle konveks çokyüzlüler eski zamanlardan beri insanların ilgilerinin odak noktasında olmuştur. Matematiksel açıdan da sahip oldukları simetrilerden dolayı önem görmüşlerdir. Çokyüzlüler, tüm güzelliklerinin ve ilginç özelliklerinin yanında anlaşılması güç şekillerdir. Bu, onların matematiksel yapılarından değil, insanların hayal edebilme güçlüklerinden kaynaklanmaktadır. Bir çokyüzlüyü göz önüne getirip ona herhangi bir açıdan zihnen bakabilmek oldukça güçtür. Özellikle yüz sayısı arttığında bu iş daha da zordur. İnsanlar çokyüzlülerle akıldan uğraşmanın çok zor olduğunun farkına varmış ve onların birer modelini yapıp, bu modeller üzerinde çalışmaya karar vermişlerdir. Böylelikle bundan daha binlerce yıl öncesinde beş, düzgün çokyüzlü modellerini yapmayı başarmışlardır. Britanya Adaları'nda yapılan arkeolojik kazılarda Platon'dan bin yıl öncesine ait, taştan yapılmış beş düzgün çokyüzlü modelleri bulunmuştur. Günümüzde de birçok kişi çokyüzlü modelleriyle uğraşmaktadır. Dünyada birçok öğretmen, öğrencilerinin el becerilerini geliştirmelerini sağlamak için onlardan kendi başlarına düzgün on ikiyüzlü ya da yirmi yüzlü modelleri yapmalarını istemektedir. Ayrıca çokyüzlü modelleri yapma işini bir hobi hatta bunun da ötesinde bir sanat olarak görenler de vardır. M.J. Wenninger, *Polyhedron Models* adlı kitabında, kendi yapımı olan çokyüzlü modellerin birer resmini ve her biri hakkında verdiği çeşitli bilgileri toplamıştır. Wenninger, kartondan yaptığı modellerin her biri için ortalama sekiz saat harcadığını söylemiştir.

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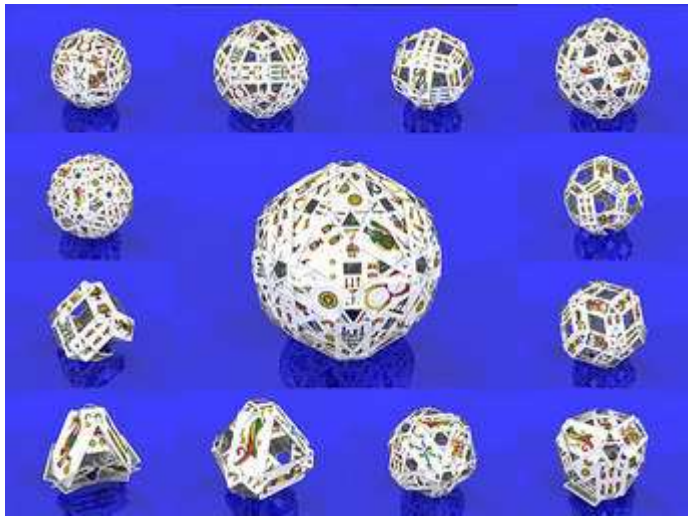


Şekil 1: Çokyüzlü Modelleri

Konveks çokyüzlülerde bir sınıflama; Arşimed Cisimleri, Katalan Cisimler ve Platonik cisimleri olarak yapılabilir. Tüm ayrıt ve yüzleri eş olan konveks çokyüzlülere düzgün çokyüzlü, eş olmayanlara ise düzgün olmayan çokyüzlü denilmektedir. Düzgün çokyüzlüler yalnızca beş tanedir ve Platonik cisimler olarak adlandırılır. Arşimet cisimler ve Katalan cisimleri ise düzgün olmayan çokyüzlülerdir. Katalan cisimlerinin tüm yüzleri aynı tip çokgenlerden oluşmaktayken, Arşimed cisimlerinin yüzleri iki veya daha fazla farklı tipten oluşabilir. Katalan cisimleri 1865 yılında Belçikalı matematikçi Eugene Catalan tarafından tanımlandıklarından Katalan cisimleri olarak isimlendirilmişlerdir . Arşimed kendi adındaki Arşimed cisimlerinden hiç bahsetmese de Pappus, 'Koleksiyon ' adlı eserinin beşinci kitabında 'Arşimed cisimlerinin' bilinen ilk tanımını vermiş ve bu isimle adlandırmıştır.

Minkowski geometrisi, eliptik ve hiperbolik geometriden farklı sonlu boyutlu Öklidyen olmayan bir geometridir. Ayrıca Space-time Minkowskian geometrisinden de farklıdır. Örneğin Öklidyen uzaydaki alışılmış kürenin yerine alınan "yeni birim küre" değişebilen bir genel simetrik konveks kümedir, ve paralellik aksiyomu geçerli olmasına rağmen, Pisagor teoremi geçerli değildir. Minkowski geometrisinde, doğrular, noktalar ve düzlemler Öklidyen geometrisi ile hemen hemen aynıdır, açı ölçümü de aynı yolla yapılır, tek fark ise uzaklığın tüm yönlerde aynı olmamasıdır. Bu farklılık da alınan metrikle ilgili kavramların değişmesine neden olmaktadır. Metrik geometri alanında yapılan birçok çalışmada Minkowski geometrileri ile çokyüzlüler arasında yakın ilişkiler gözlenmiştir. Örneğin taksi geometrinin birim küresi, bir

katalan cisim olan düzgün sekiz yüzlüdür. Minkowski geometrilerin birim kürelerinin genel simetrik konveks kümeler olması, birim küresi çokyüzlüler olan metriklerin araştırılması problemini doğurmuştur. Böylece birim küreleri konveks çokyüzlüleri verecek şekilde metrikler ve böylece bunlara ait bazı geometriler bulunmuştur (Gelişgen & Can, 2016:25),(Gelişgen & Çolak, 2015:3). Elde edilen geometrilerin sahip oldukları geometrik özelliklerin incelenmesi önemli bir konudur.

Bu çalışmada katalan bir cisim olan Tetrakis Hexahedronu birim küre kabul eden metrikle elde edilen geometrinin bazı özellikleri incelenmiştir. Bu geometri öncelikle düzlemde, sonrasında da 3-boyutlu uzayda geometrik yapısı bakımından incelenmiş, her iki durum için bulunan bazı uzaklık formüllerine değinilmiştir.

TETRAKİS HEXAHEDRON DÜZLEMİ

Tanım: $\mathcal{P}$, elemanlarına noktalar denilen bir küme ve $\mathcal{L}$ doğrular olarak isimlendirilen, $\mathcal{P}$ nin boş kümeden farklı alt kümelerinin bir kolleksiyonu olmak üzere $\mathcal{A} = \{\mathcal{P}, \mathcal{L}\}$, aşağıdaki özellikleri sağlıyorsa bir soyut geometri olarak isimlendirilir;

i) Her $A, B \in \mathcal{A}$ iki noktası için $A \in l$ ve $B \in l$ olacak şekilde bir $l \in \mathcal{L}$ vardır.

ii) Her doğrunun en az iki noktası vardır.

Tanım: $\mathcal{A} = \{\mathcal{P}, \mathcal{L}\}$ soyut geometri olmak üzere aşağıdaki aksiyomları sağlarsa $\mathcal{A} = \{\mathcal{P}, \mathcal{L}\}$ ye bir incidence geometri denir.

i) $\mathcal{L}$ deki her farklı iki nokta bir tek doğru üzerindedir.

ii) Hepsi aynı doğru üzerinde bulunmayan üç $A, B, C \in \mathcal{P}$ noktası vardır.

Tanım: $\mathcal{P}$ kümesi üzerindeki bir *uzaklık fonksiyonu*, her $P, Q \in \mathcal{P}$ için

i) $d(P, Q) > 0$

ii) $d(P, Q) = 0$ *olması için gerek ve yeter koşul* $P = Q$ *olmasıdır*

iii) $d(P, Q) = d(Q, P)$

özelliklerini sağlayan bir $d: \mathcal{P} \times \mathcal{P} \to \mathbb{R}$ fonksiyonudur.

Tanım: l, bir $\{\mathcal{P}, \mathcal{L}\}$ incidence geometrisinde bir doğru olsun. $\mathcal{P}$ üzerinde bir d uzaklık fonksiyonunun var olduğunu varsayalım. Aşağıdaki özelliklere sahip bir $\phi : \lambda \to \mathbb{R}$ fonksiyonuna l için bir cetveldir denir.

i) ϕ fonksiyonu 1-1 ve örtendir,

ii) l üzerindeki her $P,\ Q$ nokta çifti için,

$$|f(\mathrm{P}) - f(\mathrm{Q})| = d(P,Q) \tag{1}$$

dır.

(1) eşitliğine cetvel denklemi ve $f(\mathrm{P})$ ye de P nin f *ye bağlı koordinatı* denir.

Tanım: d uzaklık fonksiyonuna sahip bir $\{\mathcal{P},\mathcal{L}\}$ bir incidence geometrisinde her $l \in \mathcal{L}$doğrusu bir cetvele sahipse $\{\mathcal{P},\mathcal{L}\}$ incidence geometrisi *Cetvel Aksiyomunu* sağlar. Bu durumda $\mathcal{M} = \{\mathcal{P},\mathcal{L},\mathrm{d}\}$ sistemine bir metrik geometri denir.

Minkowski geometrisi, Öklid geometrisindeki noktalar kümesi $\mathcal{P}$, Öklid geometrisindeki doğrular kümesi $\mathcal{L}$ ve herhangi bir uzaklık fonksiyonu d olmak üzere, $\{\mathcal{P},\mathcal{L},\mathrm{d}\}$ metrik geometrisidir. O halde Minkowski geometrileri Öklidyen nokta ve doğrularla inşa edilen metrik geometrilerdir.

Tanım: $P_1 = (x_1, y_1)$ ve $P_2 = (x_2, y_2)\mathbb{R}^2$de iki farklı nokta olsun.

$$d_{TH}(P_1,P_2) = (\sqrt{3}-1)(|x_1 - x_2| + |y_1 - y_2|) + \left(2-\sqrt{3}\right)max\{|x_1 - x_2|, |y_1 - y_2|\}$$

şeklinde tanımlanan d_{TH}: $\mathbb{R}^2 \times \mathbb{R}^2 \to [0,\infty)$ uzaklık fonksiyonuna Tetrakis Hexahedron uzaklık fonksiyonu denir.

Teorem: $P = (x_1, y_1),\ Q = (x_2, y_2)$ kartezyen düzlemde herhangi iki nokta ve

$$d_{TH}(P,Q) = \left(\sqrt{3}-1\right)(|x_1 - x_2| + |y_1 - y_2|) + \left(2-\sqrt{3}\right)max\{|x_1 - x_2|, |y_1 - y_2|\}$$

biçiminde tanımlı olmak üzere, kartezyen düzlem $d_{TH} : \mathbb{R}^2 \times \mathbb{R}^2 \to [0,\infty)$ uzaklık fonksiyonu ile birlikte bir metrik geometridir.

İspat: $\{\mathcal{P},\mathcal{L},d_{TH}\}$ sisteminin metrik geometri olduğu gösterilmelidir. Bu sistemdeki nokta ve doğrular Öklidyen düzlemin nokta ve doğrularıdır. Böylece $l,\ \mathbb{R}^2_{TH}$ da herhangi bir doğru olsun. Kartezyen düzlemin d_{TH} ile birlikte bir metrik olduğunu göstermek için her l doğrusu için bir cetvel bulunmalıdır. R^2_{TH} ın doğruları $L_a = \{(x,y) \in \mathbb{R}^2_{TH} | x = a\}$ ve $L_{m,b} = \{(x,y) \in \mathbb{R}^2_{TH} | y = mx + b\}$biçiminde iki tip doğru kümesinden oluşmaktadır.

1. Durum: $l = L_a$biçiminde dikey bir doğru olsun. $P \in L_a$ve $P = (x, y)$ için $f_{TH}(P) = f_{TH}(a, y) = y$ şeklinde tanımlansın. f_{TH} fonksiyonunun bire-bir, örten olduğu ve cetvel aksiyomunu sağladığı gösterilmelidir.

i) Düzlemde $P = (x_1, y_1)$ ve $Q = (x_2, y_2)$ noktalarından geçen doğru $l = L_a$ olsun. Bu durumda $x_1 = x_2 = a$ olup $P = (a, y_1)$, $Q = (a, y_2)$ biçimindedir. $f_{TH}(P) = f_{TH}(Q)$ ise $y_1 = y_2$ olup $P = Q$ elde edilir. Dolayısıyla f_{TH} fonksiyonu bire-birdir.

ii) $\forall t \in R$ için $f_{TH}(a, t) = t$ ve $(a, t) \in l$ olduğundan f_{TH} fonksiyonu örtendir.

iii) Cetvel aksiyomunun sağlanması için $|f_{TH}(P) - f_{TH}(Q)| = d_{TH}(P, Q)$ olmalıdır. $f_{TH}(P) = y_1$ ve $f_{TH}(Q) = y_2$ olduğundan $|f_{TH}(P) - f_{TH}(Q)| = |y_1 - y_2|$ dir. Ayrıca

$$\begin{aligned} d_{TH}(P, Q) &= (\sqrt{3} - 1)\max\{|x_1 - x_2| + |y_1 - y_2|\} + \\ &\quad (2 - \sqrt{3})max\{|x_1 - x_2|, |y_1 - y_2|\} \\ &= (\sqrt{3} - 1)\max\{|a - a| + |y_1 - y_2|\} + \\ &\quad (2 - \sqrt{3})max\{|a - a|, |y_1 - y_2|\} \\ &= (\sqrt{3} - 1)\max\{|y_1 - y_2|\} + (2 - \sqrt{3})max\{|y_1 - y_2|\} \\ &= |y_1 - y_2| = |f_{TH}(P) - f_{TH}(Q)| \end{aligned}$$

olup f_{TH} fonksiyonu cetvel aksiyomunu sağlar.

2. Durum: $l = L_{m,b}$ biçiminde bir doğru olsun.

$$f_{TH}(x, y) = \begin{cases} \left(1 + \left(\sqrt{3} - 1\right)|m|\right)x & , |m| \leq 1 \\ \left(\left(\sqrt{3} - 1\right) + |m|\right)x & , |m| \geq 1 \end{cases}$$

fonksiyonu tanımlansın. f_{TH}fonksiyonunun bire-bir, örten olduğu ve cetvel aksiyomunu sağladığı gösterilmelidir.

Alt durum 1: $l = L_{m,b}$ve $|m| \leq 1$ iken f_{TH} fonksiyonunun bire-bir, örten olduğu ve cetvel aksiyomunu sağladığı gösterilmelidir.

i) Düzlemde $P = (x_1, y_1)$ve $Q = (x_2, y_2)$ noktalarından geçen doğru $l = L_{m,b}$ve $|m| \leq 1$ olsun. Bu durumda $f_{TH}(P) = \left(1 + \left(\sqrt{3} - 1\right)|m|\right)x_1$ ve $f_{TH}(Q) = \left(1 + \left(\sqrt{3} - 1\right)|m|\right)x_2$ dir. $f_{TH}(P) = f_{TH}(Q)$ ise $x_1 = x_2$ elde edilir. $y_1 = mx_1 + b$ ve $y_1 = mx_2 + b$ olduğundan $y_2 = mx_2 + b = y_1$ olup $P = Q$ bulunur, böylece f_{TH} fonksiyonu bire-birdir.

ii) $f_{TH}(x,y) = \left(1 + \left(\sqrt{3}-1\right)|m|\right)x$ ve $(x,y) \in l$ olduğundan f_{TH} fonksiyonu örtendir.

iii) Cetvel aksiyomunun sağlanması için
$|f_{TH}(P) - f_{TH}(Q)| = d_{TH}(P,Q)$ olmalıdır.

$|f_{TH}(P) - f_{TH}(Q)| = \left(1 + \left(\sqrt{3}-1\right)|m|\right)|x_1 - x_2|$dir. Ayrıca

$$\begin{aligned} d_{TH}(P,Q) &= (\sqrt{3}-1)\max\{|x_1-x_2| + |y_1-y_2|\} \\ &\quad + \left(2-\sqrt{3}\right)max\{|x_1-x_2|, |y_1-y_2|\} \\ &= (\sqrt{3}-1)\max\{|x_1-x_2| + |(mx_1+b)-(mx_1+b)|\} \\ &\quad + \left(2-\sqrt{3}\right)max\{\{|x_1-x_2| + |(mx_1+b)-(mx_1+b)|\}\} \\ &= (\sqrt{3}-1)\max\{|x_1-x_2| + |m||x_1-x_2|\} \\ &\quad + \left(2-\sqrt{3}\right)max\{|x_1-x_2|, |m||x_1-x_2|\} \\ &= \left(\sqrt{3}-1\right)|x_1-x_2| + \left(\sqrt{3}-1\right)|m||x_1-x_2| \\ &\quad + \left(2-\sqrt{3}\right)|x_1-x_2|(|m| \le 1) \\ &= (1 + \left(\sqrt{3}-1\right)|m|)|x_1-x_2| \end{aligned}$$

olduğundan $|f_{TH}(P) - f_{TH}(Q)| = d_{TH}(P,Q)$ elde edilir. Böylece f_{TH} fonksiyonu cetvel aksiyomunu sağlar.

Alt durum 2 : $l = L_{m,b}$ve $|m| \geq 1$ iken f_{TH} fonksiyonunun bire-bir, örten olduğu ve cetvel aksiyomunu sağladığı gösterilmelidir.

i) Düzlemde $P = (x_1, y_1)$ ve $Q = (x_2, y_2)$ noktalarından geçen doğru $l = L_{m,b}$ ve $|m| \geq 1$ olsun. Bu durumda $f_{TH}(P) = \left(\left(\sqrt{3}-1\right)|m|\right)x_1$ ve $f_{TH}(Q) = \left(\left(\sqrt{3}-1\right)|m|\right)x_2$ dir. $f_{TH}(P) = f_{TH}(Q)$ ise $x_1 = x_2$ elde edilir. $y_1 = mx_1 + b$ ve $y_1 = mx_2 + b$ olduğundan $y_2 = mx_2 + b = y_1$ olup $P = Q$ bulunur, böylece f_{TH} fonksiyonu bire-birdir.

ii) $f_{TH}(x,y) = \left(\left(\sqrt{3}-1\right)|m|\right)x$ ve $(x,y) \in l$ olduğundan f_{TH} fonksiyonu örtendir.

iii) Cetvel aksiyomunun sağlanması için
$|f_{TH}(P) - f_{TH}(Q)| = d_{TH}(P,Q)$ olmalıdır.

$|f_{TH}(P) - f_{TH}(Q)| = \left(\left(\sqrt{3}-1\right)|m|\right)|x_1 - x_2|$ dir. Ayrıca

$d_{TH}(P,Q) = (\sqrt{3}-1)\max\{|x_1-x_2| + |y_1-y_2|\} + \left(2-\sqrt{3}\right)max\{|x_1 - x_2|, |y_1-y_2|\}$

$$= (\sqrt{3}-1)\max\{|x_1-x_2| + |(mx_1+b)-(mx_1+b)|\}$$
$$+\left(2-\sqrt{3}\right)max\{\{|x_1-x_2| + |(mx_1+b)-(mx_1+b)|\}\}$$
$$= (\sqrt{3}-1)\max\{|x_1-x_2| + |m||x_1-x_2|\}$$
$$+\left(2-\sqrt{3}\right)max\{|x_1-x_2|, |m||x_1-x_2|\}$$
$$= (\sqrt{3}-1)|x_1-x_2| + (\sqrt{3}-1)|m||x_1-x_2|$$
$$+\left(2-\sqrt{3}\right)|m||x_1-x_2| \qquad (|m| \geq 1)$$
$$=\left(\left(\sqrt{3}-1\right)|m|\right)|x_1-x_2|$$

olup $|f_{TH}(P) - f_{TH}(Q)| = d_{TH}(P,Q)$ elde edilir. Böylece f_{TH} fonksiyonu cetvel aksiyomunu sağlar. Böylece kartezyen düzlem $d_{TH} : \mathbb{R}^2 \times \mathbb{R}^2 \to [0,\infty)$ uzaklık fonksiyonu ile birlikte bir metrik geometridir.

Teorem : $\mathbb{R}^2_{TH}, d_{TH}$ metriği ile donatılmış kartezyen düzlemi göstermek üzere, $\mathbb{R}^2_{TH}$ da bir $P = (x_0, y_0)$ noktasının $ax + by + c = 0$ denklemi ile verilen bir l doğrusuna uzaklığı

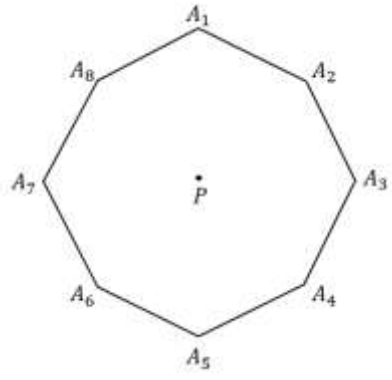
$$d_{TH}(P,l) = \frac{|ax_0 + by_0 + c|}{max\left\{|a|, |b|, \frac{|a+b|}{\sqrt{3}}, \frac{|a+b|}{\sqrt{3}}\right\}}$$

biçimindedir.

İspat: $\mathbb{R}^2_{TH}$ da $P = (x_0, y_0)$ merkezli, r yarıçaplı bir çember

$$Ç = \left\{(x,y)\middle| \begin{matrix} \left(\sqrt{3}-1\right)(|x-x_0| + |y-y_0|) \\ +\left(2-\sqrt{3}\right)max\{|x-x_0|, |y-y_0|\} = r, x, y \in \mathbb{R}, r \in \mathbb{R}^+ \end{matrix}\right\}$$

kümesi ile tanımlıdır.

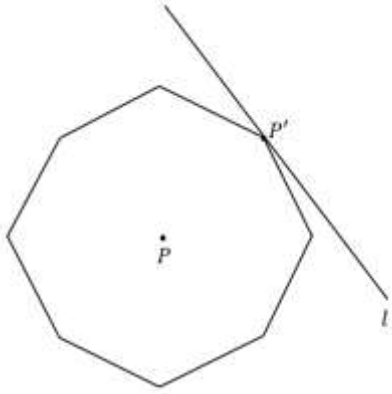


Şekil 2: Tetrakis Hexahedron Çemberi

Bu çemberin köşe noktalarının koordinatları;

$A_1 = (x_0, y_0 + r), A_2 = \left(x_0 + \frac{r}{\sqrt{3}}, y_0 + \frac{r}{\sqrt{3}}\right), A_3 = (x_0 + r, y_0),$
$A_4 = \left(x_0 + \frac{r}{\sqrt{3}}, y_0 - \frac{r}{\sqrt{3}}\right), A_5 = (x_0, y_0 - r), A_4 = \left(x_0 - \frac{r}{\sqrt{3}}, y_0 - \frac{r}{\sqrt{3}}\right),$
$A_7 = (x_0 - r, y_0), A_8 = \left(x_0 - \frac{r}{\sqrt{3}}, y_0 + \frac{r}{\sqrt{3}}\right)$ dir.

$P = (x_0, y_0)$ noktasının $ax + by + c = 0$ denklemi ile verilen bir l doğrusuna uzaklığını bulmak için P merkezli Tetrakis Hexahedron çemberinin yavaşça büyütüldüğü düşünülebilir. Böylece l doğrusunun P noktasına en yakın noktası, bu çember ile yarıçapı büyütülürken ilk kesiştiği noktası olur. Çember ile l doğrusu ilk olarak çemberin bir köşe noktasında kesişeceklerdir. Böylece ilk kesişme anında çemberin yarıçapı r olmak üzere, çemberin bir köşe noktasından geçen l' doğrusu ile l doğrusunun kesişim noktası P' ise, P noktasının l doğrusuna uzaklığı P noktasının P' noktasına uzaklığına eşittir.



Şekil 3: Tetrakis Hexahedron Çemberi İle Doğrunun Kesişme Durumu

Böylece örneğin P noktası ile A_2 köşesinden geçen doğrunun denklemi $y = x - x_0 - y_0$ dır.

$$ax+by+c=0$$

$$y= x - x_0 - y_0$$

denklem sistemi çözülürse

$$x = \frac{bx_0 - by_0 - c}{a+b} \text{ ve } y = \frac{-ax_0 + ay_0 - c}{a+b}$$

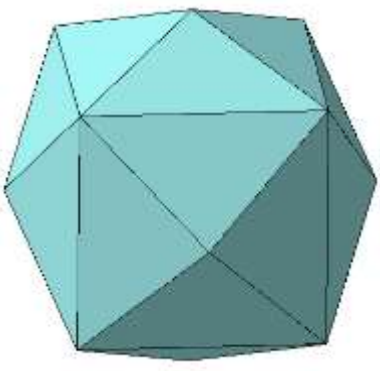
bulunur. Böylece $P' = (x, y) = \left(\frac{bx_0 - by_0 - c}{a+b}, \frac{-ax_0 + ay_0 - c}{a+b}\right)$ ile $P = (x_0, y_0)$ noktası arasındaki Tetrakis Hexahedron uzaklığı

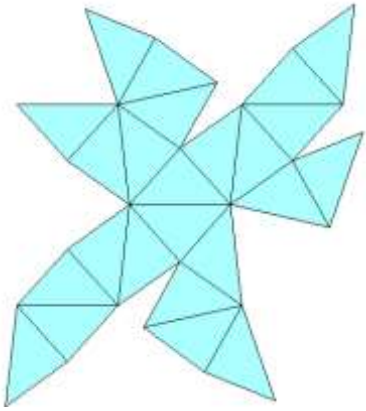
$$d_{TH}(P,P') =$$
$$(\sqrt{3}-1)\left(\left|\frac{bx_0-by_0-c}{a+b}-x_0\right|+\left|\frac{-ax_0+ay_0-c}{a+b}-y_0\right|\right)$$
$$+(2-\sqrt{3})max\left\{\left|\frac{bx_0-by_0-c}{a+b}-x_0\right|,\left|\frac{-ax_0+ay_0-c}{a+b}-y_0\right|\right\}$$
$$=\sqrt{3}\left|\frac{ax_0+by_0+c}{a+b}\right|$$

olarak bulunur. Diğer köşe noktaları ile benzer hesaplamalar yapılarak istenilen formül elde edilir.

TETRAKİS HEXAHEDRON UZAYI

TetrakisHexahedron, ikiz kenar üçgenlerin birleşmesinden oluşan 24 yüzlü bir şekildir. Toplamda 14 köşe ve 36 ayrıta sahiptir. Tetrakishexahedron 48 tane simetriye sahip olup, simetri grubu O{h} ile gösterilir. En küçük kenar uzunluğu 1 birim olup, yüzey alanı $S=(9/8)\sqrt{2}br^2$, hacmi ise $V=((32)/9)\ br^3$ tür. Tetrakis Hexahedron, kübün yüzeyi üzerine bir nokta eklenerek oluşmuş kübün kleetopu olan bir yapıdır.

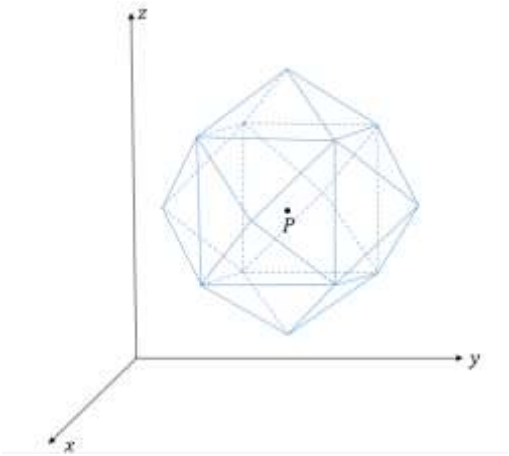




Şekil 4 : Tetrakis Hexahedron Şekil 5 : Tetrakis Hexahedronun Açılımı

Merkezil Tetrakis Hexahedron un köşe noktalarını isimlendirmek gerekirse koordinatları şu şekildedir;

$A_0 = (0,0,r)$,$A_1 = (0,0,-r)$, $A_2 = (r,0,0)$, $A_3 = (-r,0,0)$,
$A_4 = (0,r,0)$, $A_5 = (0,-r,0)$, $A_6 = \left(\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r\right)$,
$A_7 = \left(\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r,-\frac{\sqrt{3}}{3}r\right)$, $A_8 = \left(\frac{\sqrt{3}}{3}r,-\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r\right)$,
$A_9 = \left(\frac{\sqrt{3}}{3}r,-\frac{\sqrt{3}}{3}r,-\frac{\sqrt{3}}{3}r\right)$, $A_{10} = \left(-\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r,\frac{\sqrt{3}}{3}r\right)$,
$A_{11} = \left(-\frac{1}{\sqrt{3}}r,\frac{1}{\sqrt{3}}r,-\frac{1}{\sqrt{3}}r\right)$, $A_{12} = (-\frac{1}{\sqrt{3}}r,-\frac{1}{\sqrt{3}}r,\frac{1}{\sqrt{3}}r)$,
$A_{13} = (-\frac{1}{\sqrt{3}}r,-\frac{1}{\sqrt{3}}r,-\frac{1}{\sqrt{3}}r)$.



Şekil 6 : TetrakisHexahedron Uzayında P Merkezli Bir Küre

Birim küresi Tetrakis Hexahedron olan uzaklık fonksiyonu d_{TH} ile gösterilir ve aşağıdaki gibi tanımlanır.

Tanım: $P_1 = (x_1, y_1, z_1)$ ve $P_2 = (x_2, y_2, z_2)$ $\mathbb{R}^3$ te iki farklı nokta olsun.

$$d_{TH}(P_1,P_2) = (\sqrt{3}-1)max\begin{Bmatrix}|x_1-x_2|+|y_1-y_2|,\\ |x_1-x_2|+|z_1-z_2|,\\ |y_1-y_2|+|z_1-z_2|\end{Bmatrix} + \left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_1-x_2|,\\ |y_1-y_2|,\\ |z_1-z_2|\end{Bmatrix}$$

şeklinde tanımlanan d_{TH}: $\mathbb{R}^3$x $\mathbb{R}^3 \to [0,\infty)$ fonksiyonuna analitik 3-uzayda P_1 ve P_2 noktaları arasındaki Tetrakis Hexahedron uzaklık fonksiyonu denir.

Ayrıca $\alpha,\beta,\gamma \in \mathbb{R}$ olmak üzere

$$mid(\alpha,\beta,\gamma) = \alpha+\beta+\gamma-max\{\alpha,\beta,\gamma\}-min\{\alpha,\beta,\gamma\}$$

şeklinde tanımlanan $mid: \mathbb{R}\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ fonksiyonuna orta değer fonksiyonu adı verilir.

Buna göre $P_1=(x_1,y_1,z_1)$ ve $P_2=(x_2,y_2,z_2)$ $\mathbb{R}^3$ te iki nokta olmak üzere P_1 ve P_2 noktaları arasındaki Tetrakis Hexahedron uzaklığı

$$d_{TH}(P_1,P_2) = max\{|x_1-x_2|,|y_1-y_2|,|z_1-z_2|\} + (\sqrt{3}-1)\text{mid}\{|x_1-x_2|,|y_1-y_2|,|z_1-z_2|\}$$

şeklinde de verilebilir.

Yardımcı Teorem: $P_1=(x_1,y_1,z_1)$ ve $P_2=(x_2,y_2,z_2)$ $\mathbb{R}^3$ te iki farklı nokta olsun. Bu takdirde

$$d_{TH}(P_1,P_2) \geq \left(\sqrt{3}-1\right)(|x_1-x_2|+|y_1-y_2|)+\left(2-\sqrt{3}\right)|x_1-x_2| = |x_1-x_2|+\left(\sqrt{3}-1\right)|y_1-y_2|$$

$$d_{TH}(P_1,P_2) \geq \left(\sqrt{3}-1\right)(|x_1-x_2|+|y_1-y_2|)+\left(2-\sqrt{3}\right)|y_1-y_2| = |y_1-y_2|+\left(\sqrt{3}-1\right)|x_1-x_2|$$

$$d_{TH}(P_1,P_2) \geq \left(\sqrt{3}-1\right)(|x_1-x_2|+|z_1-z_2|)+\left(2-\sqrt{3}\right)|x_1-x_2| = |x_1-x_2|+\left(\sqrt{3}-1\right)|z_1-z_2|$$

$$d_{TH}(P_1,P_2) \geq \left(\sqrt{3}-1\right)(|x_1-x_2|+|z_1-z_2|)+\left(2-\sqrt{3}\right)|z_1-z_2| = |z_1-z_2|+\left(\sqrt{3}-1\right)|x_1-x_2|$$

$$d_{TH}(P_1,P_2) \geq \left(\sqrt{3}-1\right)(|y_1-y_2|+|z_1-z_2|)+\left(2-\sqrt{3}\right)|y_1-y_2| = |y_1-y_2|+\left(\sqrt{3}-1\right)|z_1-z_2|$$

$$d_{TH}(P_1,P_2) \geq (\sqrt{3}-1)(|y_1-y_2|+|z_1-z_2|)+(2-\sqrt{3})|z_1-z_2|$$
$$= |z_1-z_2|+(\sqrt{3}-1)|y_1-y_2|$$

olur.

İspat : $P_1=(x_1,y_1,z_1)$ ve $P_2=(x_2,y_2,z_2)$ analitik 3-uzayda iki nokta olsun. $a=max\{|x_1-x_2|,|y_1-y_2|,|z_1-z_2|\}$, $b=\text{mid}\{|x_1-x_2|,|y_1-y_2|,|z_1-z_2|\}$ ve $c=\min\{|x_1-x_2|,|y_1-y_2|,|z_1-z_2|\}$ olmak üzere $a\geq b\geq c$ ve $1\geq(\sqrt{3}-1)$ dir. O halde yardımcı teoremin ifadesinin ispatı açıktır. Açıkça,

$$a+(\sqrt{3}-1)b \geq a+(\sqrt{3}-1)b$$
$$a+(\sqrt{3}-1)b \geq b+(\sqrt{3}-1)a$$
$$a+(\sqrt{3}-1)b \geq a+(\sqrt{3}-1)c$$
$$a+(\sqrt{3}-1)b \geq b+(\sqrt{3}-1)c$$
$$a+(\sqrt{3}-1)b \geq c+(\sqrt{3}-1)a$$
$$a+(\sqrt{3}-1)b \geq c+(\sqrt{3}-1)b$$

olur.

Önerme : Analitik 3-uzayda d_{TH} uzaklık fonksiyonu metriktir ve bu metrikle donatılmış uzayın birim küresi Tetrakis Hexahedrondur

İspat : d_{TH}:$\mathbb{R}^3$x $\mathbb{R}^3\rightarrow[0,\infty)$ tetrakis hexahedron uzaklık fonksiyonu ve $P_1=(x_1,y_1,z_1)$, $P_2=(x_2,y_2,z_2)$, $P_3=(x_3,y_3,z_3)$ $\mathbb{R}^3$ te üç farklı nokta olsun. d_{TH} uzaklık fonksiyonu her P_1, P_2, $P_3\in\mathbb{R}^3$ için aşağıdaki aksiyomları sağlıyorsa $\mathbb{R}^3$ de bir metriktir.

M1) $d_{TH}(P_1,P_2)\geq 0$ ve $d_{TH}(P_1,P_2)=0\Leftrightarrow P_1=P_2$,

M2) $d_{TH}(P_1,P_2)=d_{TH}(P_2,P_1)$

M3) $d_{TH}(P_1,P_3)\leq d_{TH}(P_1,P_2)+d_{TH}(P_2,P_3)$

M1) Mutlak değer daima pozitif ya da sıfır olacağından mutlak değerlerin toplamının maksimum ya da minimumu da pozitiftir ya da sıfırdır. Böylece

$$d_{TH}(P_1,P_2)=(\sqrt{3}-1)max\begin{Bmatrix}|x_1-x_2|+|y_1-y_2|,\\|x_1-x_2|+|z_1-z_2|,\\|y_1-y_2|+|z_1-z_2|\end{Bmatrix}$$
$$+(2-\sqrt{3})max\begin{Bmatrix}|x_1-x_2|,\\|y_1-y_2|,\\|z_1-z_2|\end{Bmatrix}\geq 0$$

olur. Yani d_{TH} pozitif tanımlıdır ya da sıfırdır. Eğer,

$$d_{TH}(P_1,P_2) = \left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_1-x_2|+|y_1-y_2|,\\ |x_1-x_2|+|z_1-z_2|,\\ |y_1-y_2|+|z_1-z_2|\end{Bmatrix}$$

$$+\left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_1-x_2|,\\ |y_1-y_2|,\\ |z_1-z_2|\end{Bmatrix}=0$$

ise $|x_1-x_2|=0$, $|y_1-y_2|=0$ ve $|z_1-z_2|=0$ olup $x_1=x_2$, $y_1=y_2$ ve $z_1=z_2$ olur. Böylece $P_1=P_2$ bulunur.

Tersine $P_1=P_2 \Rightarrow |x_1-x_2|=0, |y_1-y_2|=0, |z_1-z_2|=0 \Longrightarrow d_{TH}(P_1,P_2)=0$ olur.

M2) Mutlak değer tanımından dolayı $|x_1-x_2|=|x_2-x_1|$, $|y_1-y_2|=|y_2-y_1|$ ve $|z_1-z_2|=|z_2-z_1|$ olacağından açıkça $d_{TH}(P_1,P_2)=d_{TH}(P_2,P_1)$ olur.

M3) $P_1=(x_1,y_1,z_1)$, $P_2=(x_2,y_2,z_2)$ ve $P_3=(x_3,y_3,z_3)\in\mathbb{R}^3$ olsun.

$$d_{TH}(P_1,P_3) \le d_{TH}(P_1,P_2)+d_{TH}(P_2,P_3)$$

olduğu gösterilmelidir.

$$d_{TH}(P_1,P_3) = \left(\sqrt{3}-1\right)max\{|x_3-x_1|+|y_3-y_1|, |x_3-x_1| + |z_3-z_1|, |y_3-y_1|+|z_3-z_1|\} + \left(2-\sqrt{3}\right)max\{|x_3-x_1|, |y_3-y_1|, |z_3-z_1|\}$$

$$= \left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_3-x_2+x_2-x_1|+|y_3-y_2+y_2-y_1|,\\ |x_3-x_2+x_2-x_1|+|z_3-z_2+z_2-z_1|,\\ |y_3-y_2+y_2-y_1|+|z_3-z_2+z_2-z_1|\end{Bmatrix}$$

$$\le \left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_3-x_2|+|x_2-x_1|+|y_3-y_2|+|y_2-y_1|,\\ |x_3-x_2|+|x_2-x_1|+|z_3-z_2|+|z_2-z_1|,\\ |y_3-y_2|+|y_2-y_1|+|z_3-z_2|+|z_2-z_1|\end{Bmatrix}$$

$$+\left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_3-x_2|+|x_2-x_1|\\ |y_3-y_2|+|y_2-y_1|\\ |z_3-z_2|+|z_2-z_1|\end{Bmatrix}$$

$$\le \left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_1-x_2|+|y_1-y_2|,\\ |x_1-x_2|+|z_1-z_2|,\\ |y_1-y_2|+|z_1-z_2|\end{Bmatrix} + \left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_1-x_2|\\ |y_1-y_2|\\ |z_1-z_2|\end{Bmatrix}$$

$$+\left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_2-x_3|+|y_2-y_3|,\\|x_2-x_3|+|z_2-z_3|,\\|y_2-y_3|+|z_2-z_3|\end{Bmatrix}+\ \left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_2-x_3|\\|y_2-y_3|\\|z_2-z_3|\end{Bmatrix}$$
$$=X$$

dir. Burada yardımcı teoremden dolayı

$$d_{TH}(P_1,P_2)\geq\left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_1-x_2|+|y_1-y_2|,\\|x_1-x_2|+|z_1-z_2|,\\|y_1-y_2|+|z_1-z_2|\end{Bmatrix}$$
$$+\ \left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_1-x_2|\\|y_1-y_2|\\|z_1-z_2|\end{Bmatrix}$$

$$d_{TH}(P_2,P_3)\geq\left(\sqrt{3}-1\right)max\begin{Bmatrix}|x_2-x_3|+|y_2-y_3|,\\|x_2-x_3|+|z_2-z_3|,\\|y_2-y_3|+|z_2-z_3|\end{Bmatrix}$$
$$+\ \left(2-\sqrt{3}\right)max\begin{Bmatrix}|x_2-x_3|\\|y_2-y_3|\\|z_2-z_3|\end{Bmatrix}$$

olur. O halde $X\leq d_{TH}(P_1,P_2)+d_{TH}(P_2,P_3)$ bulunur. Buna göre

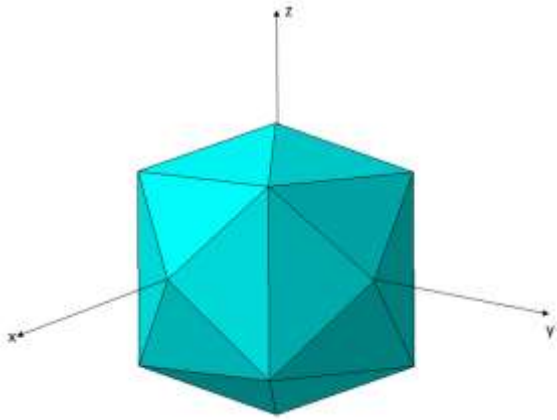
$$d_{TH}(P_1,P_3)\leq d_{TH}(P_1,P_2)+d_{TH}(P_2,P_3)$$

elde edilir. Yani tetrakis hexahedron uzaklık fonksiyonu üçgen eşitsizliğini sağlar.

Sonuç olarak tetrakis hexahedron uzaklık fonksiyonu metriktir. Ayrıca bu metrikle döşenmiş analitik 3-uzayda orijinden 1 birim tetrakis hexahedron uzaklığındaki noktaların kümesi

$$S_{TH}$$
$$=\left\{(x,y,z)\middle|\left(\sqrt{3}-1\right)max\begin{Bmatrix}|x|+|y|,\\|x|+|z|,\\|y|+|z|\end{Bmatrix}+\left(2-\sqrt{3}\right)max\begin{Bmatrix}|x|\\|y|\\|z|\end{Bmatrix}=1\right\}$$

olup bu noktaların geometrik yeri şekilde görüldüğü üzere tetrakis hexahedrondur.



Şekil 7 : Tetrakis Hexahedron Uzayında Birim Küre

Yardımcı Teorem: P_1 ve P_2 den geçen doğru l olsun.(p,q,r), l doğrusunun doğrultu vektörü ve

$$\mu = \frac{(p^2,q^2.r^2)^{\frac{1}{2}}}{(\sqrt{3}-1)\max\{|p|+|q|,|p|+|r|,|q|+|r|\}+(2-\sqrt{3})max\{|p|,|q|,|r|\}}$$

olmak üzere

$$d_E(P_1,P_2)=\mu d_{TH}(P_1,P_2)$$

dir.

İspat: $P_1=(x_1,y_1,z_1)$ ve $P_2=(x_2,y_2,z_2)$ $\mathbb{R}^3$ de herhangi iki nokta olsun. P_1 ve P_2 den geçen l doğrusunun standart denklemi $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=\lambda$ biçiminde ve

$(x_2-x_1,y_2-y_1,z_2-z_1)=(p,q,r)$ olduğundan

$$\frac{d_E(P_1,P_2)}{d_{TH}(P_1,P_2)} = \frac{(p^2,q^2.r^2)^{\frac{1}{2}}}{\sqrt{3}-1)\max\{|p|+|q|,|p|+|r|,|q|+|r|\}+(2-\sqrt{3})max\{|p|,|q|,|r|\}}$$

dir. Buradan

$$\mu = \frac{(p^2,q^2.r^2)^{\frac{1}{2}}}{(\sqrt{3}-1)\max\{|p|+|q|,|p|+|r|,|q|+|r|\}+(2-\sqrt{3})max\{|p|,|q|,|r|\}}$$

denilirse

$$d_E(P_1, P_2) = \mu d_{TH}(P_1, P_2)$$

elde edilir.

Sonuç: Herhangi bir doğru boyunca $d_{TH}(P_1, P_2)$ uzaklığı aynı doğru boyunca Öklidyen uzaklığın pozitif bir katıdır.

Sonuç: P_1, P_2 ve X, $\mathbb{R}^3$ de herhangi doğrudaş üç nokta ise $d_E(X, P_1) = d_E(X, P_2)$ olması için gerek ve yeter koşul $d_{TH}(X, P_1) = d_{TH}(X, P_2)$ olmasıdır.

Sonuç: P_1, P_2 ve X, $\mathbb{R}^3$ de herhangi doğrudaş farklı üç nokta ise

$$\frac{d_E(X, P_1)}{d_E(X, P_2)} = \frac{d_{TH}(X, P_1)}{d_{TH}(X, P_2)}$$

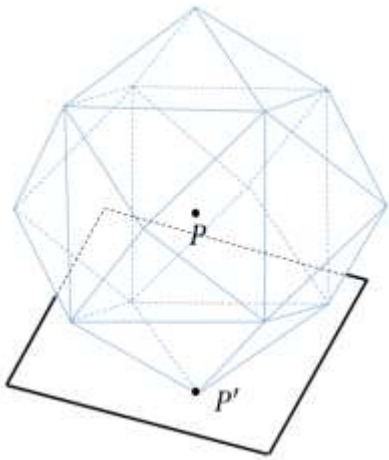
olur. Yani herhangi bir doğru boyunca Öklidyen ve Tetrakis Hexahedron uzaklıklarının oranı aynıdır.

Teorem: $\mathbb{R}^3_{TH}$,d_{TH} metriği ile donatılmış analitik 3-uzayı göstermek üzere $\mathbb{R}^3_{TH}$ da bir $P = (x_0, y_0, z_0)$ noktasının bir $\mathcal{P} = \mathrm{Ax} + \mathrm{By} + \mathrm{Cz} + \mathrm{D} = 0$ düzlemine olan Tetrakis Hexahedron uzaklığı

$$d_{TH}(P, \mathcal{P}) = min \left\{ \begin{matrix} \frac{\sqrt{3}|Ax_0+By_0+Cz_0+D|}{max\{|A+B+C|,|-A+B+C|,|A-B+C|,|A+B-C|\}} \\ \frac{|Ax_0+By_0+Cz_0+D|}{max\{|A|,|B|,|C|\}} \end{matrix} \right\}$$

ile hesaplanır.

İspat: Öklidyen uzayda sabit bir P noktasından eşit uzaklıkta bulunan noktaların kümesi bir küredir. $\mathbb{R}^3_{TH}$ de bir küre Tetrakis Hexahedron dur. Merkezinde P noktası bulunan bir kürenin yavaşça genişletildiği düşünülürse bu küre ile $\mathcal{P}$ düzleminin kesiştikleri ilk nokta P noktasına düzlemin en yakın noktası olacaktır. P merkezli Tetrakis Hexahedronun yarıçapı büyütülürse $\mathcal{P}$ düzlemi ile ilk kesişme noktası bir köşe noktasında olur. P noktası ile bir köşe noktasından geçen doğrunun doğrultusuna $(\mathrm{p}, \mathrm{q}, \mathrm{r})$ denilirse $(\mathrm{p}, \mathrm{q}, \mathrm{r}) \in \{(1,0,0), (0,1,0), (0,0,1), (1,1,1), (-1,1,1), (1,-1,1), (1,1,-1)\}$ dir. Böylece P noktasından geçen $(\mathrm{p}, \mathrm{q}, \mathrm{r})$ doğrultulu doğru ile, $\mathcal{P}$ düzleminin kesiştiği noktayla P noktası arasındaki Tetrakis Hexahedron uzaklığı, P noktası ile $\mathcal{P}$ düzlemi arasındaki Tetrakis Hexahedron uzaklığına eşittir.



Şekil 8: Tetrakis Hexahedron Uzayında Küre İle Düzlemin Kesişme Durumu

Böylece $P = (x_0, y_0, z_0)$ ve $\mathrm{P} = \mathrm{Ax} + \mathrm{By} + \mathrm{Cz} + \mathrm{D} = 0$ olsun. P noktasından geçen $(\mathrm{p}, \mathrm{q}, \mathrm{r})$ doğrultu doğru

$$\frac{x - x_0}{p} = \frac{y - y_0}{q} = \frac{z - z_0}{r} = \lambda$$

standart denklemine sahiptir. Bu doğru üzerindeki herhangi bir P' noktasının koordinatları $x = x_0 + \lambda p$, $y = y_0 + \lambda q$ ve $z = z_0 + \lambda r$ olmak üzere $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ biçimindedir. Bu nokta aynı zamanda düzlemin de üzerinde olacağından düzlem denklemini sağlar. Böylece

$$A(x_0 + \lambda p) + B(y_0 + \lambda q) + C(z_0 + \lambda r) + \mathrm{D} = 0$$

denkleminden

$$\lambda = -\frac{Ax_0 + By_0 + Cz_0 + D}{Ap + Bq + Cr}$$

bulunur. Ayrıca

$$|x - x_0| = |x_0 + \lambda p - x_0| = |p| \frac{|Ax_0 + By_0 + Cz_0 + D|}{|Ap + Bq + Cr|}$$

$$|y - y_0| = |y_0 + \lambda q - y_0| = |q| \frac{|Ax_0 + By_0 + Cz_0 + D|}{|Ap + Bq + Cr|}$$

$$|z - z_0| = |z_0 + \lambda r - z_0| = |r| \frac{|Ax_0 + By_0 + Cz_0 + D|}{|Ap + Bq + Cr|}$$

dir. Böylece

$$d_{TH}(P,P') = (\sqrt{3}-1)\max\begin{Bmatrix}|x-x_0|+|y-y_0|,\\ |x-x_0|+|z-z_0|,\\ |y-y_0|+|z-z_0|\end{Bmatrix} + (2-\sqrt{3})\max\{|x-x_0|,|y-y_0|,|z-z_0|\}$$

$$= (\sqrt{3} - 1)\max\begin{Bmatrix}|p|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|}+|q|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|},\\ |p|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|}+|r|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|},\\ |q|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|}+|r|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|},\end{Bmatrix}$$

$$+ (2-\sqrt{3})\max\begin{Bmatrix}|p|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|},\\ |q|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|},\\ |r|\frac{|Ax_0+By_0+Cz_0+D|}{|Ap+Bq+Cr|}\end{Bmatrix}$$

dir. Örneğin $(p,q,r) = (1,0,0)$ olsun. Bu durumda P noktasının Ƥ düzlemine uzaklığı;

$$d_{TH}(P,Ƥ) = (\sqrt{3}-1)max\begin{Bmatrix}|1|\frac{|Ax_0+By_0+Cz_0+D|}{|A|}+|0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|},\\ |1|\frac{|Ax_0+By_0+Cz_0+D|}{|A|}+|0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|},\\ |0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|}+|0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|},\end{Bmatrix}$$

$$+(2-\sqrt{3})max\begin{Bmatrix}|1|\frac{|Ax_0+By_0+Cz_0+D|}{|A|},\\ |0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|},\\ |0|\frac{|Ax_0+By_0+Cz_0+D|}{|A|}\end{Bmatrix}$$

$$=(\sqrt{3}-1)\frac{|Ax_0+By_0+Cz_0+D|}{|A|} + (2-\sqrt{3})\frac{|Ax_0+By_0+Cz_0+D|}{|A|}$$

$$=\frac{|Ax_0+By_0+Cz_0+D|}{|A|}$$

olarak elde edilir. Diğer doğrultular için de benzer hesaplamalarla istenilen formül bulunur.

Teorem: $\mathbb{R}^3_{TH}$ da bir$P=(x_0,y_0,z_0)$ noktasının $l=\frac{x-a}{p}=\frac{y-b}{q}=\frac{z-c}{r}$ doğrusuna Tetrakis Hexahedron uzaklığı

$$\alpha_1 = \left|\frac{(q+r)(x_0-a)-\left[p+\left(\sqrt{3}-1\right)r\right](y_0-b)+\left[-p+\left(\sqrt{3}-1\right)q\right](z_0-c)}{q+r}\right|$$

$$\alpha_2 = \left|\frac{(q+r)(x_0-a)-\left[p-\left(\sqrt{3}-1\right)r\right](y_0-b)+\left[-p-\left(\sqrt{3}-1\right)q\right](z_0-c)}{q+r}\right|$$

$$\alpha_3 = \left|\frac{(q-r)(x_0-a)+\left[-p+\left(\sqrt{3}-1\right)r\right](y_0-b)+\left[p-\left(\sqrt{3}-1\right)q\right](z_0-c)}{q-r}\right|$$

$$\alpha_4 = \left|\frac{(q-r)(x_0-a)-\left[p+\left(\sqrt{3}-1\right)r\right](y_0-b)+\left[p+\left(\sqrt{3}-1\right)q\right](z_0-c)}{q-r}\right|$$

$$\alpha_5 = \left|\frac{\left[q+\left(\sqrt{3}-1\right)r\right](x_0-a)-(p-r)(y_0-b)-\left[\left(\sqrt{3}-1\right)p+q\right](z_0-c)}{p-r}\right|$$

$$\alpha_6 = \left|\frac{\left[q-\left(\sqrt{3}-1\right)r\right](x_0-a)-(p-r)(y_0-b)+\left[\left(\sqrt{3}-1\right)p-q\right](z_0-c)}{p-r}\right|$$

$$\alpha_7 = \left|\frac{\left[q-\left(\sqrt{3}-1\right)r\right](x_0-a)-(p+r)(y_0-b)+\left[\left(\sqrt{3}-1\right)p+q\right](z_0-c)}{p+r}\right|$$

$$\alpha_8 = \left|\frac{\left[q+\left(\sqrt{3}-1\right)r\right](x_0-a)-(p+r)(y_0-b)-\left[\left(\sqrt{3}-1\right)p-q\right](z_0-c)}{p+r}\right|$$

$$\alpha_9 = \left|\frac{\left[\left(\sqrt{3}-1\right)q+r\right](x_0-a)-\left[\left(\sqrt{3}-1\right)p+r\right](y_0-b)-(p-q)(z_0-c)}{p-q}\right|$$

$$\alpha_{10} = \left|\frac{\left[\left(\sqrt{3}-1\right)q-r\right](x_0-a)-\left[\left(\sqrt{3}-1\right)p-r\right](y_0-b)+(p-q)(z_0-c)}{p-q}\right|$$

$$\alpha_{11} = \left|\frac{\left[\left(\sqrt{3}-1\right)q+r\right](x_0-a)-\left[\left(\sqrt{3}-1\right)p-r\right](y_0-b)-(p+q)(z_0-c)}{p+q}\right|$$

$$\alpha_{12} = \left|\frac{\left[\left(\sqrt{3}-1\right)q-r\right](x_0-a)-\left[\left(\sqrt{3}-1\right)p+r\right](y_0-b)+(p+q)(z_0-c)}{p+q}\right|$$

ve

$i=\{1,2,\dots,12\}$ için

$$\Delta=\begin{cases} q+r,\ \alpha_1\geq\alpha_i\ \ veya\ \ \alpha_2\geq\alpha_i \\ q-r,\ \alpha_3\geq\alpha_i\ \ veya\ \ \alpha_4\geq\alpha_i \\ p-r,\ \alpha_5\geq\alpha_i\ \ veya\ \ \alpha_6\geq\alpha_i \\ p+r,\ \alpha_7\geq\alpha_i\ \ veya\ \ \alpha_8\geq\alpha_i \\ p-q,\ \alpha_9\geq\alpha_i\ \ veya\ \ \alpha_{10}\geq\alpha_i \\ p+q,\ \alpha_{11}\geq\alpha_i\ \ veya\ \ \alpha_{12}\geq\alpha_i \end{cases}$$

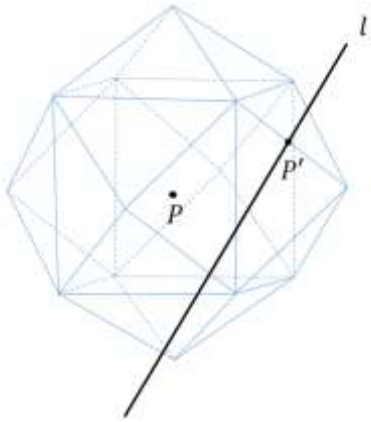
olmak üzere;

$$d_{TH}(P,\mathrm{l})=\frac{max\{\alpha_i\}}{\Delta}$$

ile hesaplanır.

İspat: $\mathbb{R}^3_{TH}$ da bir $P=(x_0,y_0,z_0)$ noktasının $\frac{\mathrm{x-a}}{\mathrm{p}}=\frac{\mathrm{y-b}}{\mathrm{q}}=\frac{\mathrm{z-c}}{\mathrm{r}}$ denklemi ile verilen bir l doğrusuna Tetrakis Hexahedron uzaklığını bulmak için, merkezi P olanTetrakis Hexahedron küresi düşünülebilir. P- merkezli bu kürenin yarıçapı büyütülürse küre ile doğrunun kesişme noktası ile P noktası arasındaki uzaklık, P noktasının l doğrusuna uzaklığı ile aynı olacaktır. Tetrakis Hexahedron küresi ile doğru, kürenin bir köşesinde, ayrıtında ya da yüzeyinde kesişecektir. Her köşe bir ayrıt üzerinde olduğundan ve yüzeyler ayrıtlarla sınırlandırıldığından ayrıtlarla doğrunun kesişme durumunun incelenmesi yeterlidir. Kesişme anında kürenin yarıçapı k olmak üzere bazı köşe noktalarının koordinatları;

$P_1=(x_0+k,y_0,z_0)$, $P_2=(x_0-k,y_0,z_0)$, $P_3=(x_0,y_0+k,z_0)$,
$P_4=(x_0,y_0-k,z_0)$, $P_5=(x_0,y_0,z_0+k)$, $P_6=(x_0,y_0,z_0-k)$,
$P_7=\left(x_0+\frac{1}{\sqrt{3}}k,y_0+\frac{1}{\sqrt{3}}k,z_0+\frac{1}{\sqrt{3}}k\right)$,
$P_8=\left(x_0-\frac{1}{\sqrt{3}}k,y_0+\frac{1}{\sqrt{3}}k,z_0-\frac{1}{\sqrt{3}}k\right)$,
$P_9=\left(x_0+\frac{1}{\sqrt{3}}k,y_0-\frac{1}{\sqrt{3}}k,z_0-\frac{1}{\sqrt{3}}k\right)$,
$P_{10}=\left(x_0-\frac{1}{\sqrt{3}}k,y_0-\frac{1}{\sqrt{3}}k,z_0-\frac{1}{\sqrt{3}}k\right)$ dır.



Şekil 9: P Merkezli Tetrakis Hexahedron Küresinin Bir l Doğrusu İle Kesişme Durumu

Böylece Tetrakis Hexahedron küresi ile l doğrusu

i) doğrultusu $\Delta_1 = \left\{ \begin{matrix} (1,1,1-\sqrt{3}), (-1,1,1-\sqrt{3}), \\ (1,-1,1-\sqrt{3}), (-1,-1,1-\sqrt{3}) \end{matrix} \right\}$ kümesinin bir elemanı ve P_5 veya P_6 dan geçen bir doğru,

ii) doğrultusu $\Delta_2 = \left\{ \begin{matrix} (1,1-\sqrt{3},1), (1,1-\sqrt{3},-1), \\ (-1,1-\sqrt{3},1), (-1,1-\sqrt{3},-1) \end{matrix} \right\}$ kümesinin bir elemanı ve P_3 veya P_4 ten geçen bir doğru,

iii) doğrultusu $\Delta_3 = \left\{ \begin{matrix} (1-\sqrt{3},1,1), (1-\sqrt{3},-1,1), \\ (1-\sqrt{3},1,-1), (1-\sqrt{3},-1,-1) \end{matrix} \right\}$ kümesinin bir elemanı ve P_1 veya P_2 den geçen bir doğru,

veya

iv) doğrultusu $\Delta_4 = \{(1,0,0), (0,1,0), (0,0,1)\}$ kümesinin bir elemanı ve P_7, P_8, P_9 veya P_{10} dan geçen bir doğru üzerindeki bir ayrıtta kesişecektir.

Örneğin $P_1 = (x_0 + k, y_0, z_0)$ noktasından geçen ve $(1-\sqrt{3}, -1, 1)$ doğrultulu ayrıtı göz önüne alalım. Bu durumda bu ayrıt

$$\frac{x-(x_0+k)}{1-\sqrt{3}} = \frac{y-y_0}{-1} = \frac{z-z_0}{1} = \lambda$$

standart denklemine sahip bir doğru üzerindedir. Bu ayrıt ile $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r} = \mu$ standart denklemli l doğrusunun ara kesitinin koordinatlarının (x, y, z) olduğu varsayılsın. Böylece

$(x,y,z)=\left(\left(1-\sqrt{3}\right)\lambda+x_0+k, y_0-\lambda, z_0+\lambda\right)$ ve
$(x,y,z)=(a+p\mu, b+q\mu, c+r\mu)$ olup

$$\left(1-\sqrt{3}\right)\lambda+(x_0-a)=-k+p\mu,$$
$$-\lambda+(y_0-b)=q\mu,$$
$$\lambda+(z_0-c)=r\mu$$

denklem sisteminden doğru ile Tetrakis Hexahedron küresinin kesişme anındaki küre yarıçapı olan

$$k=\left|\frac{(q+r)(x_0-a)-\left[p+\left(\sqrt{3}-1\right)r\right](y_0-b)-\left[p-\left(\sqrt{3}-1\right)q\right](z_0-c)}{q+r}\right|$$

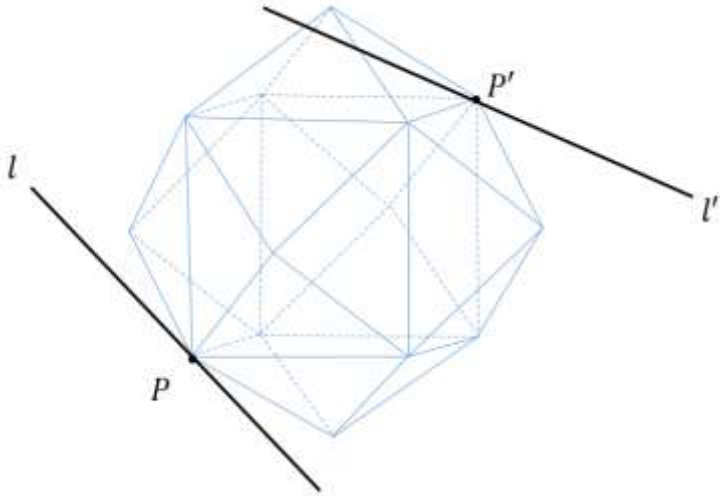
değeri elde edilir. Bu da istenildiği gibi P noktasının l doğrusuna uzaklığıdır. Benzer olarak diğer durumlar da çözülerek araştırılan formül elde edilir.

Teorem: $\mathbb{R}^3_{TH}$ da $\frac{x-a}{p}=\frac{y-b}{q}=\frac{z-c}{r}$ denklemi ile verilen bir l doğrusunun, $\frac{\mathrm{x-a'}}{\mathrm{p'}}=\frac{\mathrm{y-b'}}{\mathrm{q'}}=\frac{\mathrm{z-c'}}{\mathrm{r'}}$ denklemi ile verilen bir l' doğrusuna uzaklığı $\alpha=qr'-rq'$, $\beta=p'r-pr'$, $\gamma pq'-p'q$ olmak üzere

$$d_{TH}(l,l')=\frac{|\alpha(a-a')+\beta(b-b')+\gamma(c-c')|}{max\left\{|\alpha|,|\beta|,|\gamma|,\frac{|\alpha+\beta+\gamma|}{\sqrt{3}},\frac{|-\alpha+\beta+\gamma|}{\sqrt{3}},\frac{|\alpha-\beta+\gamma|}{\sqrt{3}},\frac{|\alpha+\beta-\gamma|}{\sqrt{3}}\right\}}$$

ile hesaplanır.

İspat: $l:\ \frac{\mathrm{x-a}}{\mathrm{p}}=\frac{\mathrm{y-b}}{\mathrm{q}}=\frac{\mathrm{z-c}}{\mathrm{r}}=\lambda$ ve $\mathrm{l'}:\frac{\mathrm{x-a'}}{\mathrm{p'}}=\frac{\mathrm{y-b'}}{\mathrm{q'}}=\frac{\mathrm{z-c'}}{\mathrm{r'}}=\mu$ olsun. Bu iki doğru birbirine paralel ise $(\mathrm{p,q,r})=(\mathrm{p',q',r'})$ dir. l üzerinde bir nokta $\mathrm{A}=(\mathrm{a,b,c})$ olup $\mathrm{d_{TH}(l,l')=d_{TH}(A,l')}$ yazılabilir. Böylece iki doğru arasındaki Tetrakis Hexahedron uzaklığı noktanın doğruya olan uzaklığı formülünden kolayca bulunabilir.



Şekil 10: Tetrakis Hexahedron Küresinin İki Doğru İle Kesişme Durumu

Bu iki doğru birbirine paralel değil ise, $pq' - p'q, pr' - p'r, qr' - q'r$ ifadelerinden en az biri sıfırdan farklıdır. Aksi taktirde bu iki doğru birbirine paralel olurdu. l ve l' üzerinde sırasıyla $P = (p\lambda + a, q\lambda + b, r\lambda + c)$ ve $P' = (p'\mu + a', q'\mu + b', r'\mu + c')$ noktaları göz önüne alınsın. Bu durumda $d_{TH}(P, P')$ değerinin minimum olması için P ve P' noktalarından geçen l" doğrusunun doğrultusu (1,0,0), (0,1,0), (0,0,1), (1,1,1), $(-1,1,1)$, $(1,-1,1)$ veya $(1,1,-1)$ olabilir. Örneğin (p,q,r)=(1,0,0) ise

$$\frac{p\lambda + a - p'\mu - a'}{1} = \frac{q\lambda + b - q'\mu - b'}{0} = \frac{r\lambda + c - r'\mu - c'}{0}$$

olup

$$\lambda = \frac{-r'(b - b') + q'(c - c')}{qr' - q'r}$$

ve

$$\mu = \frac{-r(b' - b) + q(c - c')}{qr' - q'r}$$

elde edilir. Bulunan bu λ ve μ değerleri P ve P' noktalarında yerlerine yazılıp bu iki nokta arasındaki Tetrakis Hexahedron uzaklığı hesaplanırsa;

$$d_{TH}(P, P') = \frac{|qr' - q'r||a - a'| + |p'r - pr'||b - b'| + |pq' - p'q||c - c'|}{|qr' - q'r|}$$

elde edilir. Benzer şekilde diğer doğrultular için de aynı hesaplamalar yapılarak iki doğru arasındaki Tetrakis Hexahedron uzaklığı için istenilen formül elde edilir.

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EXAMINATION OF MULTI STRUCTURES ON TOPOLOGICAL SPACES BITOPOLOGICAL SPACES AND DITOPOLOGICAL SPACES

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1. Introduction

Topology is simply geometry rendered flexible. Topology allows shapes to be changed, but without tearing them. This will be codified by open sets. Topological spaces form the broadest method in which the notion of a continuous function makes sense. We can then formulate classical and basic theorems about continuous functions in a much broader framework.

In this important branch of mathematics, multi structures such as bitopological spaces and ditopological spaces were created. Bitopological space is formulated by Kelly in 1963. Thereafter a large numbers of papers (Lane, 1967; Patty, 1967; Pervin, 1967), have been done to generalize the topological concepts into bitopological settings. In (Datta, 1972) identified the projective objects in a suitable category of bitopological spaces and generalized the category of compact Hausdorff topological spaces.

The theory of bitopological spaces have widen until today. But, the researchers have wanted to know the theory of bitopological spaces is a generalization of the theory of topological spaces, or not. The first effectual research had come from (Mukharjee, 2013). In this work it was proved that the notions introduced were not generalization of any result of topological spaces since nontrivial similar concepts in topological spaces were absurd. However, when the two topologies in a bitopological space coincide, these notions reduce to equivalent conventional concepts of topological spaces. This study is based on the research (Şenel, 2009) generally and is reproduced from this work. Apart form definitions and theorems are numbered, known concepts are mentioned in the text along with the reference (Şenel, 2009).

The previously known results as well the results obtained in

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(Dvalishvili, 1980) are applied in analysis, potential theory, general topology, and theory of ordered topological spaces. Moreover, a high level of modern knowledge of bitopological spaces theory has made it possible to introduce and study algebra of new type, the corresponding representation of which brings one to the special class of bitopological spaces in this special work. It is beyond any doubt that in the nearest future the areas of essential applications will be the theories of linear topological spaces and topological groups, algebraic and differential topologies, the homotopy theory, not to mention other fundamental areas of modern mathematics such as geometry, mathematical logic, the probability theory and many other areas, including those of applied nature.

Levelheaded discoveries are motivated by the desire to make the multi structures as accessible as possible. It seems that the relations between topological spaces and bitopological spaces emerged most clearly when the setting is quite abstract, and this (rather than a desire for mere generality) motivates the researcher's approach to construct a new topological structure on bitopological spaces. So, ditopological space is introduced by (Brown & Diker, 1998). In this work, it is established that intuitionistic topologies are mapped to ditopologies on the corresponding texture, and some notions of compactness and stability are considered. The paper of these authors (Brown, Ertürk, & Dost, 2004) which develop various fundamental aspects of the theory of ditopological texture spaces in a categorical setting and present important links with the theory of L-topological spaces. The authors begin by defining the notion of q-sets, which together with the p-sets considered earlier, enable the formulation of a powerful concept of duality. This plays an important role in the theory of direlations and difunctions, which is described in detail.

The main objective of this paper is to observe detailed examination about multi structures on topological spaces such as bitopological and ditopological spaces. It should be also noted that at present there are several hundred works dedicated to the investigation of multi structures; most of them deal with the theory of itself, but very few deal with applications.

Consideration of a set with a specific topological structure can not be sufficient to solve problems, it becomes necessary to introduce an additional structure on this set. To confirm this idea, multi structures on topological spaces such as bitopological space and ditopological space are introduced. The aim of this work is to observe detailed examination about multi structures and handle a high level of knowledge of multi topological structure theory has made it possible to introduce and study these constructive topological structures.

In many aspects of mathematics, bitopological space and ditopological space theory have a tremendous application such as the theories of linear bitopological and ditopological groups, algebraic and different multi topologies, not to mention other fundamental areas of modern mathematics including those of applied nature. From this standpoint of applications of multi topologies has no less promising prospects than the theory of topological spaces.

2. Bitopological Spaces

In this section, we recall some basic notions of bitopological space theory which may be found in (Kelly, 1963; Lane, 1967; Patty, 1967; Pervin, 1967; Şenel, 2009) for further details. Apart from definitions and theorems are numbered, known concepts are mentioned in the text along with the reference (Şenel, 2009).

Definition 2.1. (Kelly, 1963) Let X ≠ ∅, τ_1 and τ_2 be two different topologies on X. Then (X, τ_1, τ_2) is called a bitopological space (BT-space). Throughout this paper (X, τ_1, τ_2) [or simply X] denote bitopological space on which no seperation axioms are assumed unless explicitly stated.

Definition 2.2. (Şenel, 2009) A subset A of X is called $\tau_1\tau_2$-open if $A \in \tau_1 \cup \tau_2$ and the complement of $\tau_1\tau_2$ open is $\tau_1\tau_2$ closed.

Example 2.3. Let $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. The sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, are called $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$ are called $\tau_1\tau_2$ closed.

Definition 2.4. (Şenel, 2009) Let A be a subset of X. Then,

i. The $\tau_1\tau_2$-closure of A, denoted by $\tau_1\tau_2 cl(A)$, is defined by

$$(\bar{A})_{\tau_1\tau_2} = \cap\{F: A \subset F, F \text{ is a } \tau_1\tau_2 - closed\}$$

ii. The $\tau_1\tau_2$-interior of A, denoted by $\tau_1\tau_2 int(\mathrm{A})$, is defined by

$$(A)^{\circ}_{\tau_1\tau_2} = \cup\{F: A \subset F, F \text{ is a } \tau_1\tau_2 - open\}$$

Note that $(A)^{\circ}_{\tau_1\tau_2}$ is the biggest $\tau_1\tau_2$-soft open set that contained in A and $(\bar{A})_{\tau_1\tau_2}$is the smallest $\tau_1\tau_2$-soft closed set that containing A.

Theorem 2.5. (Şenel, 2009) Let (X, τ_1, τ_2) be a $BT - space$ and $A, B \subseteq X$ Then,

i. $(A)^{\circ}_{\tau_1} \subseteq (A)^{\circ}_{\tau_1\tau_2}$

ii. $(A)^{\circ}_{\tau_2} \subseteq (A)^{\circ}_{\tau_1\tau_2}$

iii. $(A)^{\circ}_{\tau_1\tau_2} \subseteq (A)^{\circ}_{\tau_1}$

iv. $(A)^{\circ}_{\tau_1\tau_2} \subseteq (A)^{\circ}_{\tau_2}$

Theorem 2.6. (Şenel, 2009) Let (X, τ_1, τ_2) be a BT-space and $A, B \subseteq X$. Then,

i. $(\overline{A \cap B})_{\tau_1\tau_2} \subseteq (\bar{A})_{\tau_1\tau_2} \cap (\bar{B})_{\tau_1\tau_2}$

ii. $(A)^{\circ}_{\tau_1\tau_2} \cup (B)^{\circ}_{\tau_1\tau_2} \subseteq (A \cup B)^{\circ}_{\tau_1\tau_2}$

Theorem 2.7. (Dvalishvili, 1980) Let (X, τ_1, τ_2) and (X, τ_1', τ_2') be two BT-spaces over the same universe X, then $(X, \tau_1 \cap \tau_2, \tau_1' \cap \tau_2')$ is a BT-space over X.

Theorem 2.8. (Dvalishvili, 1980) Let (X, τ_1, τ_2) be a BT-space. Then, the following conditions hold:

i. The universal set X and X^c are $\tau_1\tau_2 - closed$ sets.

ii. Arbitrary intersections of the $\tau_1\tau_2 - closed$ sets are $\tau_1\tau_2 - closed$.

iii. Finitely many unions of the $\tau_1\tau_2 - closed$ sets are $\tau_1\tau_2 - closed$.

Theorem 2.9. (Dvalishvili, 1980) Let (X, τ_1, τ_2) be a BT-space and $A \subseteq X$. Then, A is a

$\tau_1\tau_2$-open set if and only if $A = (A)^{\circ}_{\tau_1\tau_2}$.

Theorem 2.10. (Dvalishvili, 1980) Let (X, τ_1, τ_2) be a BT-space and $A, B \subseteq X$. Then,

i. $\left((A)^{\circ}_{\tau_1\tau_2}\right)^{\circ}_{\tau_1\tau_2} = (A)^{\circ}_{\tau_1\tau_2}$

ii. $A \tilde{\subseteq} B \Longrightarrow (A)^{\circ}_{\tau_1\tau_2} \subseteq (B)^{\circ}_{\tau_1\tau_2}$

iii. $(A)^{\circ}_{\tau_1\tau_2} \cap (B)^{\circ}_{\tau_1\tau_2} = (A \cap B)^{\circ}_{\tau_1\tau_2}$

Theorem 2.11. (Dvalishvili, 1980) Let (X, τ_1, τ_2) be a BT-space and $A \subseteq X$. Then, A is a $\tau_1\tau_2 - closed$ soft set if and only if $A = (\bar{A})^{\circ}_{\tau_1\tau_2}$.

Theorem 2.12. (Mukharjee, 2013) Let (X, τ_1, τ_2) be a $BT - space$ and $A \subseteq X$. Then,

$(A)^{\circ}_{\tau_1\tau_2} \subseteq A \subseteq (\bar{A})^{\circ}_{\tau_1\tau_2}$.

Theorem 2.13. (Mukharjee, 2013) Let (X, τ_1, τ_2) be a BT-space and $A, B \subseteq X$. Then,

i. $\overline{\left((\bar{A})^{\circ}_{\tau_1\tau_2}\right)}_{\tau_1\tau_2} = (\bar{A})_{\tau_1\tau_2}$

ii. $(\bar{A})^{c}_{\tau_1\tau_2} = (A^{c})^{\circ}_{\tau_1\tau_2}$

iii. $B \subseteq A \Longrightarrow (B)^{\circ}_{\tau_1\tau_2} \subseteq (\bar{A})_{\tau_1\tau_2}$

iv. $\overline{(A \cap B)}_{\tau_1\tau_2} \subseteq (\bar{A})_{\tau_1\tau_2} \cap (\bar{B})_{\tau_1\tau_2}$

v. $(\bar{A})_{\tau_1\tau_2} \cup (\bar{B})_{\tau_1\tau_2} = \overline{(A \cap B)}_{\tau_1\tau_2}$

Theorem 2.14. (Mukharjee, 2013) Let (X, τ_1, τ_2) be a BT-space and B $\tau_1\tau_2$ be a basis for (X, τ_1, τ_2). Then $\tau_1 \cup \tau_2$ equals the collection of all unions of elements $B\tau_1\tau_2$.

3. Ditopological Spaces

Ditopological texture spaces were introduced by L.M. Brown (Brown & Diker, 1998) as a point-based setting for the study of fuzzy sets. In this study texture spaces characterized which arise in the lattice of intuitionistic sets on a set X and gave some topological results. In (Yıldız & Özçağ, 2013) it is shown that if the complemented discrete ditopological texture space is bi-submaximal, then dicompactness implies strong dicompactness. This result remains an open problem to determine conditions for general ditopological texture spaces. In this section the general information about ditopological space is given based on the study (Yıldız & Özçağ, 2013).

Definition 3.1. (Brown & Diker, 1998) Ditopology: A dichotomous topology on $(S; \mathcal{S})$, or ditopology for short, is a pair $(\tau; \mathcal{K})$ of generally unrelated subsets τ, κ of $\mathcal{S}$ satisfying

(τ_1) $S, \emptyset \in \tau$

(τ_2) $G_1, G_2 \in \tau \Longrightarrow G_1 \cap G_2 \in \tau$

(τ_3) $G_i \in \tau, i \in I \Longrightarrow \bigvee_i G_i \in \tau,$

(κ_1) $S, \emptyset \in \kappa,$

(κ_2) $K_1, K_2 \in \kappa \Longrightarrow K_1 \cap K_2 \in \kappa,$

(κ_3) $K_i \in \kappa, i \in I \Longrightarrow \bigcap K_i \in \kappa.$

The elements of τ are called open and those of κ closed. We refer to τ as the topology and κ as the cotopology of (τ, κ).

If (τ, κ) is a ditopology on a complemented texture $(S, \mathcal{S}, \sigma)$, then we say that (τ, κ) is complemented if the equality $\kappa = \sigma(\tau)$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \mathcal{S}, \sigma, \tau, \kappa)$.

For $A \in S$ the closure clA and interior $intA$ of A are defined by the

equalities

$clA = \bigcap\{K \in \kappa | A \subseteq K\}$ and $intA = \bigvee\{G \in \tau | G \subseteq A\}$

If (τ, κ) is a complemented ditopology on $(S; \mathcal{S}, \sigma)$, then we have $\sigma(clA) = int\sigma(A)$ and $\sigma(intA) = cl\sigma(A)$.

Let $(S; \mathcal{S})$, $(T; \mathcal{T})$ be textures. In the following definition we consider $\mathcal{P}(S) \otimes \mathcal{T}$ and denote the psets and qsets by $\overline{P}_{(s,t)}$ and $\overline{Q}_{(s,t)}$ respectively. The notion of di-function is derived from that of direlation (Kelly, 1963). Now we recall the definition of difunction.

Difunction: (Brown & Diker, 1998) A difunction from $(S; \mathcal{S})$ to $(T; \mathcal{T})$ is a direlation $(f; F)$ from $(S; \mathcal{S})$ to $(T; \mathcal{T})$ satisfying the conditions:

$(DF1)$ For $s, s' \in S, P_s \nsubseteq Q_{s'} \Longrightarrow \exists t \in T$ with $f \nsubseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \nsubseteq F$.

$(DF2)$ For $t, t' \in T$ and $s \in S, f \nsubseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \nsubseteq F \Longrightarrow P_{t'} \nsubseteq Q_t$.

If $(f; F)$ is a difunction on $(S; \mathcal{S})$ to $(T; \mathcal{T})$, then $(f; F)$ is called surjective if it satisfies the condition

$SU\ R$. For For $t, t' \in T, P_t \nsubseteq Q_{t'} \Longrightarrow \exists s \in S$ with $f \nsubseteq Q_{s,t'}$ and $\overline{P}_{s,t} \nsubseteq F$.

Bicontinuity: (Yıldız & Özçağ, 2013) The difunction $(f; F) : (S_1; \mathcal{S}_1; \tau_1; \kappa_1) \to (S_2; \mathcal{S}_2; \tau_2; \kappa_2)$ is called continuous if $B \in \tau_2 \Longrightarrow F^{\leftarrow} B \in \tau_1$, cocontinuous if $B \in \kappa_2 \Longrightarrow f^{\leftarrow} B \in \kappa_1$, and bicontinuous if it is both continuous and cocontinuous.

The early works on compactness in ditopological texture spaces was begun in (Brown & Diker, 1998). Now let us recall some concepts from (Yıldız & Özçağ, 2013) which will be needed.

Let $(\tau; \kappa)$ be a ditopology on $(S; \mathcal{S})$ and take $A \in \mathcal{S}$. The family $\{G_i | i \in I\}$ is called open cover of A if $G_i \in \tau$ for all $i \in I$ and $A \subseteq \bigvee_{i \in I} G_i$. A closed cocover can be defined dually, i.e. the family $\{F_i | i \in I\}$ is called closed cocover of A if $F_i \in \kappa$ for all $i \in I$ and $\bigcap_{i \in I} F_i \subseteq A$.

Dicompactness: (Yıldız & Özçağ, 2013) Let $(\tau; \kappa)$ be a ditopology on the texture $(S; \mathcal{S})$ and $A \in \mathcal{S}$.

1) A is called compact if whenever $\{G_i | i \in I\}$ is an open cover of A, then there is a finite subset J of I with $A \subseteq \bigcup_{j \in J} G_j$. The ditopological texture space $(S; \mathcal{S}; \tau; \kappa)$ is called compact if S is compact.

2) A is called cocompact if $F_i|i \in I$ is a closed cocover of A, then there is a finite subset J of I with $\bigcap_{j\in J} F_j \subseteq A$. The ditopological texture space $(S; \mathcal{S}; \tau; \kappa)$ is called cocompact if $\emptyset$ is cocompact.

3) $(\tau; \kappa)$ is called stable if every $K \in \kappa$ with $K \neq S$ is compact.

4) $(\tau; \kappa)$ is called costable if every $G \in \tau$ with $G \neq \emptyset$ is cocompact.

Theorem 3.2. (Yıldız & Özçağ, 2013) *A difunction* $(f; F):(S_1; \mathcal{S}_1; \tau_1; \kappa_1) \to (S_2; \mathcal{S}_2; \tau_2; \kappa_2)$ is $Strongly\text{-}prebicontinuous \Rightarrow bicontinuous \Rightarrow prebicontinuous.$

Theorem 3.3. (Yıldız & Özçağ, 2013) *A difunction* $(f; F): (S_1; \mathcal{S}_1; \tau_1; \kappa_1) \to (S2; \mathcal{S}2; \tau_2; \kappa_2)$ is $Strongly\text{-}prebicontinuous \Rightarrow M - prebicontinuous \Rightarrow prebicontinuous.$

Corollary 3.4. (Yıldız & Özçağ, 2013) Let $(S_1; \mathcal{S}_1; \tau_1; \kappa_1)$, $(S_2; \mathcal{S}_2; \tau_2; \kappa_2)$ be ditopological texture spaces and $(f; F): (\mathcal{S}_1; S_1) \to (S_2; \mathcal{S}_2)$: be a surjective difunction. Then:

1) If $(S_1; \mathcal{S}_1; \tau_1; \kappa_1)$ is strongly compact and $(f; F)$ is precontinuous, then $(S_2; \mathcal{S}_2; \tau_2; \kappa_2)$ is compact.

2) If $(S_1; \mathcal{S}_1; \tau_1; \kappa_1)$ is strongly cocompact and $(f; F)$ is precontinuous, then $(S_2; \mathcal{S}_2; \tau_2; \kappa_2)$ is cocompact.

4. Conclusion

The main objective of this paper is to observe detailed examination of multi structures on topological spaces especially as bitopological and ditopological space theories. The results in this study which indicates that bitopological spaces have had a powerful in influence on the development of the theory of ditopological spaces, but it should be emphasized that a ditopology and a bitopology are conceptually different. The difference is that a bitopology consists of two separate topological structures whose interrelations we wish to study, whereas a ditopology represents a single topological structure. The main advantage of the use of a ditopology is that it does away with the need for a complement of any form. The information about multi topological structures provides an insight to applications of this work where is given a degree or grade of openness in each topological spaces.

5. Declarations

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UNIVALENT FUNCTIONS AND ITS SOME SPECIAL SUBCLASSES

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1. Introduction

In this section we will give basic definitions and theorems.

1.1 Definition. Let z_0 be a fixed point on the complex plane $\mathbb{C}$ and let r be positive real number $0 < r < \infty$, we write

$$D(z_0, r) = \{z: |z - z_0| < r\}.$$

The set $D(z_0, r)$ is called, the open disk of radius r centered at z_0 (Palka, 1991).

Specially if $z_0 = 0$ and $r = 1$, we say $D(0,1) = \mathbb{D}$ open unit disk.

1.2. Definition. Let g be a function of the two real variables x and y. We say that

$$\lim_{(x,y)\to(x_0,y_0)} g(x, y) = L$$

if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$|g(x, y) - L| < \varepsilon \text{ whenever } 0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta.$$

If $L = g(x_0, y_0)$, then $g(x, y)$ is said to be continuous at (x_0, y_0) (Ponnusamy & Silverman, 2006).

1.3. Definition. A function $f: D \to \mathbb{C}$, $D \subset \mathbb{C}$ is said to be differentiable at a point z_0 provided z_0 is an interior point of A at which the limit

$$\lim_{z\to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

exists. When this is the case we appropriate the familiar notation and terminology found in calculus, writing

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$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

and speaking of $f'(z_0)$ as the derivative of f at z_0 (Palka, 1991).

1.4.Theorem. If f is differentiable at a point $z \in \mathbb{C}$, then f is continuous at z (Ponnusamy & Silverman, 2006).

1.5.Theorem. If $f = u + iv$ is differentiable at z, then f_x, f_y exist at z and satisfy the Cauchy-Riemann equations at z:

$$f_y(z) = if_x(z)$$

or equivalently, $u_x = v_y$ and $u_y = -v_x$ (Ponnusamy & Silverman, 2006).

Now, we will give definition of analytic(or regular or holomorphic) functions which defined by different authors.

1.6. Definition. Let $f(z) = u(x, y) + iv(x, y)$ be defined in a domain D, u_x, u_y, v_x, v_y, are continuous and let $u(x, y)$ and $v(x, y)$ satisfy the Cauchy–Riemann equations

$$u_x = v_y \text{ and } u_y = -v_x$$

for all points in D. Then $f(z)$ is *analytic* in D (Ponnusamy & Silverman, 2006).

1.7. Definition. A complex-valued function f defined on an open set $D \subset \mathbb{C}$ is differentiable at $z_0 \in D$ if there exists a function Δ on D, continuous at z_0, such that

$$f(z) = f(z_0) + \Delta(z)(z - z_0)$$

holds for all $z \in D$. If f is differentiable at all points $z_0 \in D$, then we say that f is *analytic* on D (Fischer & Lieb, 2012).

1.8. Definition. Such a function f is *analytic* at z_0 if it is differentiable at every point in some neighborhood of z_0. It is one of the "miracles" of complex analysis that f must have derivatives of all orders at z_0, and that f has a Taylor series expansion

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n, \qquad a_n = f^n(z_0)/n!$$

convergent in some open disk centered at z_0 (Duren, 1983).

2. Univalent Functions

Now let's make a definition of the univalent function that is our main subject.

2.1. Definition. A function $f(z)$ is said to be univalent in a domain D if the conditions

$$f(z_1) = f(z_2), \qquad z_1, z_2 \in D$$

imply that $z_1 = z_2$. Univalent functions are called simple or schlicht by some authors (Goodman, 1983).

We are focused in univalent functions which are also analytic in B. The theory of univalent functions is so vast and complicated that certain simplifying assumptions are necessary. The most obvious one is to replace the arbitrary domain B by one that is convenient, and the most attractive selection is the open unit disk $\mathbb{D} = \{z: |z| < 1\}$.

2.2. Definition. If f is analytic and univalent in the open unit disk $\mathbb{D}$ and satisfying the conditions $f(0) = 0$ and $f'(0) = 1$ then f is called a normalized analytic univalent function. The class of all normalized analytic univalent functions is denoted by S.

The theory of univalent functions is largely concerned with the family of S. If $f \in S$, f has

$$f(z) = z + a_2 z^2 + \cdots = z + \sum_{n=2}^{\infty} a_n z^n, \qquad z \in \mathbb{D} \qquad (1)$$

the Taylor expansion.

Many authors have worked with univalent functions over the years. Several subclasses of univalent functions have been defined and some criteria have been given for these functions to belong to the subclasses. For the class of S, growth and distortion theorems and coefficient estimates studies were made.

Some of the most common subclass of univalent functions; convex and starlike functions, close to convex functions, spirallike functions, concave univalent functions and harmonic univalent functions. And there are many authors which are interested in these classes for example: Robertson (1936), Spacek (1933), Silverman & Telage (1979), Bayram & Yalçın (2017), Bayram & Altınkaya (2017), Çakmak et al (2017) etc.

Now we will give definitions and some basic properties for some of these subclasses.

2.3. Definition. P denotes the class of functions $p(z)$ analytic in $\mathbb{D}$ with

$$p(0) = 1, \quad \mathrm{Re}p(z) > 0 \quad (z \in \mathbb{D}).$$

Here it is understood that functions in class P do not need to be univalent.

S denote the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ which are regular and univalent in $|z| < 1$ and which map $|z| < 1$ onto domains $f(\mathbb{D})$. Let C, S^*, and K represent the subclasses of S where $f(\mathbb{D})$ are respectively, close-to-convex, starlike with respect to the origin, and convex. It follows that $K \subset S^* \subset C \subset S$. We will simply say "starlike" when we mean starlike with respect to the origin, and the statement " $f(z)$ is convex" will mean that the domain $f(\mathbb{D})$ is convex. The following results, which we will use repeatedly, are well known (Bernardi, 1952):

Let $f(z) \in S$. Then

$f(z) \in S^*$ if and only if $zf'(z)/f(z) \in P$;

$f(z) \in K$ if and only if $[1 + zf''(z)/f'(z)] \in P$;

$f(z) \in K$ if and only if $zf'(z) \in S^*$.

2.4. Theorem. If $f(z)$ analytic in a convex domain D and $\mathrm{Re}\{f'(z)\} > 0$ then $f(z)$ is univalent in D (Noshiro, 1934-1935, Warschawski, 1935).

Proof. Let z_1 and z_2 be distinct points in D. Then f is defined on the linear segment joining z_1 to z_2, and

$$f(z_1) - f(z_2) = \int_{z_1}^{z_2} f'(z)dz$$

$$= (z_2 - z_1) \int_0^1 f'(tz_2 + (1-t)z_1)dt$$

$$\neq 0,$$

since $\mathrm{Re}f'(z) > 0$.

A function f analytic in the unit disk is said to be close to convex if there is a convex function g such that

$$\operatorname{Re}\left\{\frac{f'(z)}{g'(z)}\right\} > 0$$

for all $z \in \mathbb{D}$. This class introduced by Kaplan (1952).

2.5. Theorem. Every close-to-convex function is univalent.

Proof. If f is close-to-convex, then $\operatorname{Re}\{f'(z)/g'(z)\} > 0$ for some convex function g. Let D be the range of g and consider the function

$$h(w) = f\left(g^{-1}(w)\right), \quad w \in D.$$

Then

$$h'(w) = \frac{f'(g^{-1}(w))}{g'(g^{-1}(w))} = \frac{f'(z)}{g'(z)}$$

so $\operatorname{Re}\{h'(w)\} > 0$ in D. Thus h is univalent, by Theorem 2.4 and so f if univalent.

In this chapter, we will focus on harmonic univalent functions.

3. Harmonic Univalent Functions

A real-valued function $u(x, y)$ is real harmonic if it satisfies Laplace's equation:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

If $u(x, y)$ and $v(x, y)$ are real harmonic, then $f(z) = u(z) + iv(z)$ is complex-valued harmonic function. It is convenient to use the complex notation $z = x + iy$, $w = u + iv$ and to write $w = f(z) = u(z) + iv(z)$.

It follows from the Cauchy-Riemann equations and from the existance of higher derivatives that every analytic function is harmonic. A pair of functions (u, v) that satisfy the Cauchy-Riemann equations is said to be a conjugate pair, and v is called the harmonic conjugate of u. Hence, $-u$ is the harmonic conjugate of v.

Therefore we can write the canonical representation

$$f = h + \bar{g}$$

of a harmonic function f in the unit disk $\mathbb{D}$ as the sum of an analytic function h and the conjugate of an analytic function g. The representation is unique.

An analytic univalent function is called a conformal mapping because it preserves angles between curves. In fact, this angle-preserving property characterizes analytic functions among all functions with continuous first partial derivatives and nonvanishing Jacobians, because it implies that the Cauchy-Riemann equations are satisfied (Duren 2004).

Closely connected are harmonic univalent mappings, which are widely known to have a wealth of applications. They arise in the modelling of many physical problems, such as in the study of fluid dynamics and elasticity problems, in the approximation theory of plates subjected to normal loading, and in the investigations of Stokes flow in the engineering and biological transport phenomena. Harmonic mappings are also important to differential geometers because these maps provide isothermal (or conformal) parameters for minimal surfaces. Indeed various properties of minimal surfaces such as the Gauss curvature are studied more effectively through planar harmonic mappings.

Although a harmonic map provides a natural generalization to studies on analytic univalent functions, surprisingly it fails to capture the interest of function theorists for quite a period of time. The defining moment came with the seminal paper by Clunie and Sheil-Small in 1984. They introduced complex analytic approach in their studies and succeeded in finding viable analogues of the classical growth and distortion theorems, covering theorems, and coefficient estimates in the general setting of planar harmonic mappings. Although there have been substantial steps forward in the studies of harmonic mappings, yet many fundamental questions and conjectures remain unresolved. There is a great expectation that the "harmonic Koebe function" will play the extremal role in many of these problems, much akin to the role played by the Koebe function in the classical theory of analytic univalent functions (Ravichandran et. al. 2014).

3.1. Remark. The square of a harmonic function need not to be harmonic. A harmonic function of an analytic function is harmonic, but an analytic function of a harmonic function need not to be harmonic. The harmonic functions do not form an algebra but the analytic function form an algebra. The inverse of a harmonic mappings need not to be harmonic. Analytic functions are preserved under compositions, but harmonic functions are not.

3.2. Definitions. Let $\mathcal{H}$ denote the family of continuous complex valued harmonic functions which are harmonic in $\mathbb{D}$ and let $\mathcal{A}$ be the subclass of $\mathcal{H}$ consisting of functions which are analytic in $\mathbb{D}$. A function harmonic in $\mathbb{D}$ may be written as $f = h + \bar{g}$; where h and g are members

of $\mathcal{A}$. We call h the analytic part and g co-analytic part of f. A necessary and sufficient condition for f to be locally univalent and sense-preserving in $\mathbb{D}$ is that $|h'(z)| > |g'(z)|$ (Clunie & Sheil-Small 1984). This shows in particular that $h'(z) \neq 0$ so there is no loss of generality in supposing that $h(0) = 0$ and $h'(0) = 1$. To this end, without loss of generality, we may write

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad \text{and} \quad g(z) = \sum_{n=1}^{\infty} b_n z^n \qquad (2)$$

Let $S\mathcal{H}$ (or $S_{\mathcal{H}}$) denote the family of functions $f = h + \bar{g}$ which are harmonic, univalent, and sense-preserving in $\mathbb{D}$ for which $f(0) = f_z(0) - 1 = 0$. Thus, $S\mathcal{H}$ contains the standard class S of analytic univalent functions. Although the analytic part h of a function $f \in S\mathcal{H}$ is locally univalent, it will become apparent that it need not be univalent. One shows easily that the sense-preserving property implies that $|b_1| < 1$. The subclass $S\mathcal{H}^0$ of $S\mathcal{H}$ consists of all functions in $S\mathcal{H}$ which have the additional property $f_{\bar{z}}(0) = 0$. In 1984 Clunie and Sheil-Small investigated the class $S\mathcal{H}$ as well as its geometric subclasses and obtained some coefficient bounds. Since then, there have been several related papers on $S\mathcal{H}$ and its subclasses. Also note that $S\mathcal{H}$ reduces to the class S of normalized analytic univalent functions in $\mathbb{D}$, if the co-analytic part of f is identically zero.

3.3. Theorem. Each function $f \in S\mathcal{H}^0$ satisfies the inequality

$$|f(z)| \geq \frac{1}{4}\frac{|z|}{(1+|z|)^2}, \qquad |z| < 1.$$

In particular, the range of f contains the disk $|w| < 1/16$ (Clunie & Sheil-Small 1984).

Now we call the Harmonic Koebe function belongs to $S\mathcal{H}^0$.

3.4. Definition. Under favorable conditions let

$$H(z) = \frac{z - \frac{1}{2}z^2 + \frac{1}{6}z^3}{(1-z)^3}, \quad G(z) = \frac{\frac{1}{2}z^2 + \frac{1}{6}z^3}{(1-z)^3}$$

and $H(0) = G(0) = 0$. $K = H + \bar{G}$ is called harmonic Koebe function (Clunie & Sheil-Small 1984).

3.5. Theorem. Let α be the supremum of $|a_2|$ among all functions $f \in S\mathcal{H}$. Then every function $f \in S\mathcal{H}^0$ satisfies the inequalities

$$\frac{1}{2\alpha}\left[1-\left(\frac{1-r}{1+r}\right)^{\alpha}\right] \le |f(z)| \le \frac{1}{2\alpha}\left[\left(\frac{1+r}{1-r}\right)^{\alpha}-1\right]$$

for $r = |z| < 1$. In particular, the range of each function $f \in S\mathcal{H}^0$ contains the disk $|w| < 1/2\alpha$ (Sheil-Small 1990).

Proof. Choosing $f = h + \bar{g} \in S\mathcal{H}$ and fixing $\xi \in \mathbb{D}$, we apply a disk automorphism to obtain the function

$$F(z) = \frac{f\left(\frac{z+\xi}{1+\bar{\xi}z}\right) - f(\xi)}{(1-|\xi|^2)h'(\xi)} = H(z) + \overline{G(z)} \in S\mathcal{H}.$$

Let

$$H(z) = z + A_2(\xi)z^2 + A_3(\xi)z^3 + \cdots$$

and if we do a simple calculation we get

$$A_2(\xi) = \frac{1}{2}\left((1-|\xi|^2)\frac{h''(\xi)}{h'(\xi)} - 2\bar{\xi}\right).$$

But $|A_2(\xi)| \le \alpha$ by definition of α, which implies that

$$\frac{2r^2-2\alpha r}{1-r^2} \le \operatorname{Re}\left\{\frac{zh''(\xi)}{h'(\xi)}\right\} \le \frac{2r^2+2\alpha r}{1-r^2}, \qquad |z| = r < 1.$$

This inequality can be recast in the form

$$\frac{2r-2\alpha}{1-r^2} \le \frac{\partial}{\partial r}\left\{\log\left|h'\left(re^{i\theta}\right)\right|\right\} \le \frac{2r+2\alpha}{1-r^2}.$$

After integration from 0 to r and exponentiation, we arrive at the estimate

$$\frac{(1-r)^{\alpha-1}}{(1+r)^{\alpha+1}} \le |h'(z)| \le \frac{(1+r)^{\alpha-1}}{(1-r)^{\alpha+1}}, \qquad |z| = r < 1.$$

Now suppose that $f = h + \bar{g} \in S\mathcal{H}^0$, so that $f + \overline{cf} \in S\mathcal{H}$ for every choice of complex constant $c \in \mathbb{D}$. Then, by what we have just proved,

$$\frac{(1-r)^{\alpha-1}}{(1+r)^{\alpha+1}} \le |h'(z) + cg'(z)| \le \frac{(1+r)^{\alpha-1}}{(1-r)^{\alpha+1}}.$$

This shows in particular that

$$\frac{(1-r)^{\alpha-1}}{(1+r)^{\alpha+1}} \leq |h'(z)| - |g'(z)|$$

and

$$|h'(z)| + |g'(z)| \leq \frac{(1+r)^{\alpha-1}}{(1-r)^{\alpha+1}}.$$

The last inequality shows that

$$|f(z)| = \left| \int_{\Gamma} \frac{\partial f}{\partial \xi} d\xi + \frac{\partial f}{\partial \bar{\xi}} d\bar{\xi} \right| \leq \int_{\Gamma} (|h'(\xi)| + |g'(\xi)|)|d\xi|$$

$$\leq \int_0^r \frac{(1+\rho)^{\alpha-1}}{(1-\rho)^{\alpha+1}} d\rho = \frac{1}{2\alpha} \left[\left(\frac{1+r}{1-r} \right)^{\alpha} - 1 \right]$$

for $|z| = r < 1$, where Γ is the radial segment from 0 to z. Next let Γ be the preimage under f of the radial segment from 0 to $f(z)$. Then

$$|f(z)| = \left| \int_{\Gamma} \frac{\partial f}{\partial \xi} d\xi + \frac{\partial f}{\partial \bar{\xi}} d\bar{\xi} \right| \geq \int_{\Gamma} (|h'(\xi)| - |g'(\xi)|)|d\xi|$$

$$\leq \int_0^r \frac{(1-\rho)^{\alpha-1}}{(1+\rho)^{\alpha+1}} d\rho = \frac{1}{2\alpha} \left[1 - \left(\frac{1-r}{1+r} \right)^{\alpha} \right],$$

which completes the proof.

4. Examples

Now we will give examples for a better understanding of the subject.

4.1. Example. Does an univalent function need to be analytic?

From the Definition 2.1 we can say that an univalent function may not be analytic. For example;

$$f(z) = z + \frac{1}{2}\bar{z}^2.$$

We want to show univalence of this function, for $z_1, z_2 \in \mathbb{D}$

$$f(z_1) = f(z_2)$$

$$z_1 + \frac{1}{2}\overline{z_1}^2 = z_2 + \frac{1}{2}\overline{z_2}^2$$

$$2(z_1 - z_2) = \bar{z_2}^2 - \bar{z_1}^2$$

$$2(z_1 - z_2) = (\bar{z_2} + \bar{z_1})(\bar{z_2} - \bar{z_1})$$

By taking the absolute values on both sides, we see that this is imposible unless $z_1 = z_2$ because $|z_1 - z_2| < 2$. Therefore $f(z)$ is univalent on $\mathbb{D}$. Now let $z = x + iy$ and we obtain

$$f(z) = x + iy + \frac{1}{2}(x - iy)^2 = x + \frac{1}{2}(x^2 - y^2) + i(y - xy)$$

here

$$u(x, y) = x + \frac{1}{2}(x^2 - y^2)$$

and

$$v(x, y) = y - xy.$$

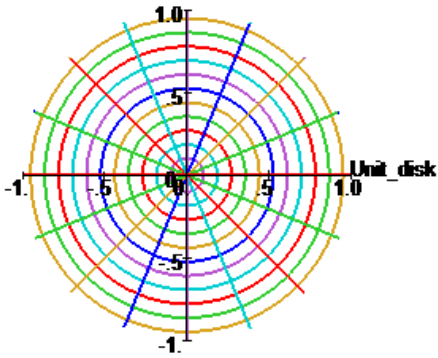
Then

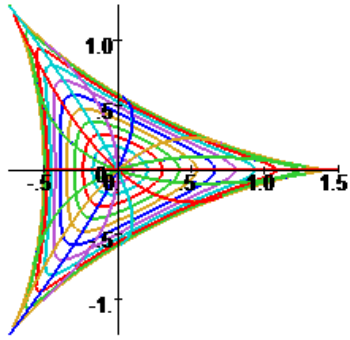
$$u_x = 1 + x = 1 - x = v_y \Longrightarrow x = 0$$

$$v_x = -y = y = -u_y \Longrightarrow y = 0$$

on $\mathbb{D}$. Only when $(x, y) = (0,0)$ Cauchy-Riemann equations satisfies. So that $f(z)$ is not analytic on $\mathbb{D}$.

Here is $\mathbb{D}$ and $f(\mathbb{D})$ images:

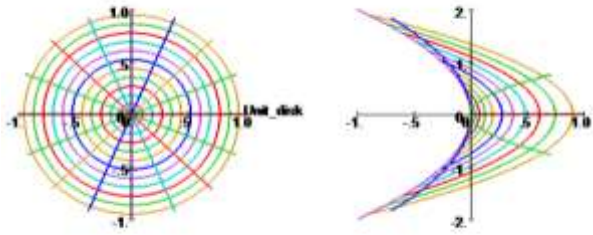




4.2. Example. Does a harmonic function need to be univalent?

Let $f(z) = z^2$, we will investigate harmonic for $z = x + iy$, $f(x + iy) = (x + iy)^2 = x^2 - y^2 + 2ixy$ then $u(x, y) = x^2 - y^2$ and $v(x, y) = 2ixy$. Therefore we obtain $u_{xx} + u_{yy} = 0$ and $v_{xx} + v_{yy} = 0$, so $f(z)$ is a harmonic function on $\overline{\mathbb{D}}$. Now let's see if $f(z)$ is univalent. To do this, accept that $f(z_1) = f(z_2)$ then ${z_1}^2 = {z_2}^2$ and we can easily see that equality is provided for two different values $z_1 = i$ and $z_2 = -i$, so $z_1 \neq z_2$. As a result $f(z)$ is not univalent in $\overline{\mathbb{D}}$.

Now we will find $f(\mathbb{D})$ image with Maple:



4.3 Exercise. Let $f(z) = z + \bar{z}$, show $f(z)$ is a harmonic function but not univalent in $\mathbb{D}$.

4.4. Example. Show the function which define in Example 4.1 is a harmonic function.

Get help from Maple13, we must show that

$$u_{xx} + u_{yy} = 0$$

and

$$v_{xx} + v_{yy} = 0.$$

```
> z:=x+I*y;
```

$$z := x + y\,I$$

```
> f(z):=z+(1/2)*(conjugate(z))^2;
```

$$f(x + y\,I) := x + y\,I + \frac{1}{2}\overline{(x + y\,I)}^2$$

```
>
value(diff(simplify(evalc(Re(f(z)))),x$2)+dif
f(simplify(evalc(Re(f(z)))),y$2));
```

0

```
>
value(diff(simplify(evalc(Im(f(z)))),x$2)+dif
f(simplify(evalc(Im(f(z)))),y$2));
```

0

4.5. Example. Show that $u = xy^3 - yx^3 + 2x$ is harmonic and find its harmonic conjugates.

Let's solve this question with Maple13.

```
> u:=x*y^3-y*x^3+2*x;
```

$$u := x\,y^3 - y\,x^3 + 2\,x$$

```
> u_x:=diff(u,x);
```

$$u_x := y^3 - 3\,y\,x^2 + 2$$

```
> u_y:=diff(u,y);
```

$$u_y := 3\,x\,y^2 - x^3$$

```
> u_xx:=diff(u_x,x);
```

$$u_xx := -6\,y\,x$$

```
> u_yy:=diff(u_y,y);
```

$$u_yy := 6\,y\,x$$

```
> u_xx+u_yy;
```

0

```
> simplify(u_xx+u_yy);
```

0

```
> v_1:=int(u_x,y);
```

$$v_1 := \frac{1}{4}y^4 - \frac{3}{2}y^2\,x^2 + 2\,y$$

```
> v_1x:=diff(v_1,x);
```

$$v_1x := -3\,x\,y^2$$

```
> v_1x:=simplify(v_1x);
```

$$v_1x := -3\,x\,y^2$$

```
> Cx:= (-u_y) - v_1x;
```

$$Cx := x^3$$

```
> Cx:=simplify(Cx);
```

$$Cx := x^3$$

```
> C:=int(Cx,x);
```

$$C := \frac{x^4}{4}$$

```
> v:=v_1+C;
```

$$v := \frac{1}{4}y^4 - \frac{3}{2}y^2x^2 + 2y + \frac{1}{4}x^4$$

```
> simplify(diff(u,x)-diff(v,y));
```

$$0$$

```
> simplify(diff(u,y)+diff(v,x));
```

$$0$$

```
> w:=u+I*v;
```

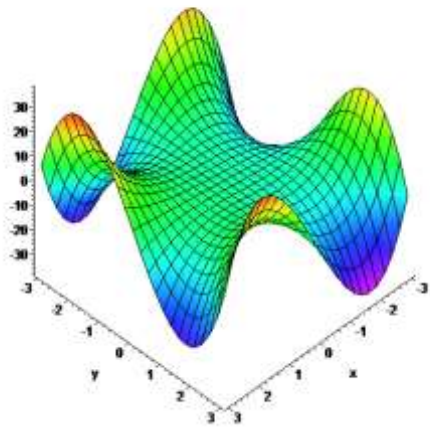
$$w := xy^3 - yx^3 + 2x + \left(\frac{1}{4}y^4 - \frac{3}{2}y^2x^2 + 2y + \frac{1}{4}x^4\right)I$$

Moreover we will show u and v part of this function.

```
> u:=(x,y)->(x*y^3-y*x^3+2*x);
```

$$u := (x, y) \to xy^3 - yx^3 + 2x$$

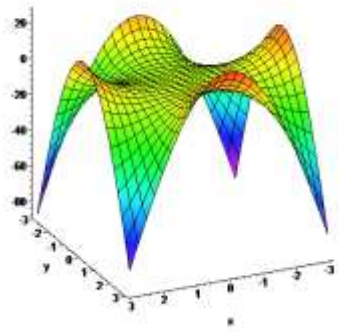
```
> plot3d(u(x,y),x=-3..3,y=-
3..3,axes=framed,shading=zhue);
```



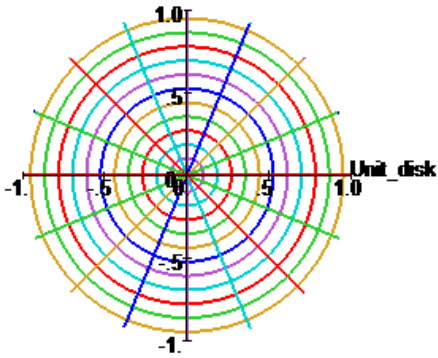
```
> v:=(x,y)->(1/4*y^4-
3/2*x^2*y^2+2*y+1/4*x^4);
```

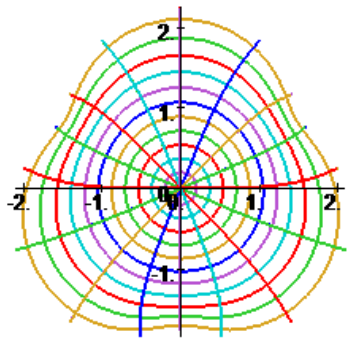
$$v := (x, y) \to \frac{1}{4}y^4 - \frac{3}{2}x^2y^2 + 2y + \frac{1}{4}x^4$$

```
> plot3d(v(x,y),x=-3..3,y=-
3..3,axes=framed,shading=zhue);
```



And we can find image the unit disk under $f = u + iv$ harmonic function.





With using this method we can solve difficult problem.

4.6. Exercise. Show the function

$$u = \mathrm{Re}\left(\frac{z+3}{z^3-1}\right)$$

is harmonic and find the harmonic conjugate of u. (Hint: In order to solve this problem, it is necessary to do a lot of heavy work and you can easily solve it like Example 4.5).

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MEDICINAL IMPORTANCE AND POTENTIAL OF SUMMER ASPHODEL (*Asphodelus aestivus* Brot.): THE COMMON WEED PLANT OF MEDITERRANEAN RANGELANDS

Fırat ALATÜRK[**] & Ahmet GÖKKUŞ[*] & Baboo ALİ[*1]

Introduction

The summer asphodel, *Asphodelus aestivus* Brot., is a perennial, herbaceous and tuberous weed plant belongs to Liliaceae family, and can reach the height up to one meter. Blooming starts in April and May, and the plant begins to dry from the second half of May due to hot and dry weather conditions. It grows again in October and November depending on the rainfall and temperature of autumn. In Mediterranean rangelands, the plant continues to exist under hot climate, but it shows to develop during rainy and cool weather conditions of the year. A geophyte that upper soil parts become dry and its tubers remain alive in hot and drought seasons. This weed plant has the ability to cover a huge area, especially the degraded areas, in the environments where it conforms by multiplying with its tubers and seeds (Sawidis et. al., 2005). The tuber continues its growth if the environmental conditions are favorable and the plant is healthy. Since the upper ground portions of weed plant, which is mostly found in natural rangelands, are not consumed by the animals even they are green due to their toxic structure (glycosides such as steroidal saponins), and their competitiveness does not weaken in grazing conditions. Although, for this reason, it is not encountered in animals due to this plant poisonings, it is still a significant weed in the Mediterranean rangeland areas of our country. Poisoning rarely occurs because animals do not graze by recognizing this weed plant. But, on the other hand, it is consumed by the animals because of the dryness of their leaves and the decreasing rate of glycoside (Dahlgren et al., 1985), and in this case, the plants are not affected because the dead tissues are grazed by the animals.

Summer asphodel competes with other plants in rangelands with high nutritional value, thus causing undesirable changes in vegetation cover by suppressing their growth and gaining importance with their medicinal properties, even this aspect of plant attracts more attention. In its

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structure, due to the presence of c-glycoside, it is used in the treatment of many diseases (Dahlgren et al., 1985; Çalış et al., 2006). Therefore, in this chapter, the distribution, density, medicinal properties, biopesticide importance and biological control of this plant by *Capsodes infuscatus* (Hemiptera: Miridae) have been tried to be emphasized.

Distribution

Summer asphodel is a common and invasive weed plant in arid and semi-arid Mediterranean ecosystems (Margaris, 1984; Eltez, 1995). Some studies show that this weed plant has also spread in certain parts of North Africa (Le Houerou, 1979). These ecosystems, commonly named as "asphodel deserts", are defined as the last phase of deterioration in the Mediterranean Regions (Ayyad & Hilmy, 1974; Le Houerou, 1981). In Turkey, the growing belt of this weed plant starts from Sanliurfa Province, especially, in the rangelands of Aegean and Marmara Regions and partially found in forest ranges (Davis, 1965).

This weed plant is commonly found in heavy grazing, frequent fire catching and erosion affected areas (Naveh, 1973; Pantis & Stamou, 1991). Fires, heavy grazing and population abundance of certain insect species promote the spread of summer asphodel (Pantis & Mardiris, 1992; Rhizopoulou et al., 1997). This situation shows not only the adaptation of this weed plant under Mediterranean climatic conditions but also the ability to cope with difficult weather circumstances (Sawidis et al., 2005).

Since the percentage of fruit-forming flowers (Schuster et al., 1993) in the plant is low (between 8-37%), it has enough seed to spread the plant easily by producing between 180-280 number (Eltez, 1995) or 400-600 number (Sawidis et al., 2008) of flowers. However, the effect of the tubers on the spread of this weed plant is already limited.

The ratio of *Asphodelus* spp. reach up to 10-50% especially, in Aegean and Eastern Mediterranean rangelands (Bilgir, 1961; Uygun et al., 1994). The number of plants reaches up to 10 plants/m^2 in the rangelands of Aegean Region (Eltez, 1995), but this number reaches up to 33 plants/m^2 in those rangelands found in Canakkale Province (Alatürk et al., 2017). The number of tubers per plant varies between 30-50 (Eltez, 1995).

Under natural conditions, the growth of this weed plant is suppressed according to the variation of climatic conditions and the presence of various species of insect pests, especially *Capsodes infuscatus*, one of the natural enemies of this weed plant. However, these natural factors do not always occur, that is why, a large-scale and continuous suppression of this weed plant cannot be discussed. Flower buds cannot be formed even

if the flower stalks are produced in case of rainy weather of spring, and accordingly the soil moisture level increases, even though, the formed flowers become dry and fall into the ground within a short period of time before reaching to its maturity. Consequently, the distribution of this weed plant is not possible.

Beside of this, the development and distribution of the plant can be suppressed by different insect species i.e., *Capsodes infuscatus* (Hemiptera: Miridae) is one of the biological control agents of summer asphodel. As the population density of *C. infuscatus* increases, the number of flowers and the nectar production of summer asphodel decreases significantly. *C. infuscatus* may deteriorate the success of pollination of summer asphodel plants by reducing the amount of nectar in the flowers (Samocha & Sternberg, 2010). *C. infuscatus* can attack the plants heavily and prevent their fruit formation where summer asphodel is found in more or less frequent, but some of the fruit could be able to hold seeds into it where the plants are found in dense amount. Therefore, there is an inverse relationship between plant frequency and insect damage (Izhaki et al., 1996). A ratio of 13% of the summer asphodel plants did not form any flower stalks because of the damage of *C. infuscatus* insect population. Flower stalks emerged only in 20% of *A. aestivus* plants, but they could not complete their growth and development and dried out within a short period of time (Eltez, 1995). However, in our country, the population density of *C. infuscatus* on the summer asphodel has only been determined under natural conditions, and not being manipulated as biological control tool directly into *A. aestivus* dominated rangelands.

Characteristic Features of Plant

The upper soil parts of summer asphodel possess a poisonous and bitter taste. Intense nerve pigmentation along with severe neural syndrome, yellowish-brown color changes in the brain and spinal cord and emergence of pigment particles of yellowish-brown color in the cytoplasm of large neurons are observed in the sheep those grazed on the leaves of *A. aestivus* plant (Birincioğlu et al., 2005). Due to the above mentioned causes, animals live in the rangelands are able to recognize easily the summer asphodel and not prefer to graze on that weed plant. In resulting, especially in heavy grazing rangelands, other plant species cannot find an opportunity to regenerate themselves under grazing pressure, but the summer asphodel easily shows an increase in a non-competitive environment with its seeds and tubers. Animals only consume the leaves when the plant becomes dry. Nonetheless, the plant is not affected under such circumstances. Because the sub soil parts of the plant remain alive even in dry season in summer and animals could not graze on these parts of the summer asphodel.

Summer asphodel is a plant that has resistant to fire as well as grazing. This weed plant has also adapted to shallow soils. The risk of fires (periodic fires) is high in the Mediterranean shrub formation (Trabaud, 1997; Syphard et al., 2009) and the soil is mostly superficial, especially due to erosion on rough terrain (Raclot et al., 2016). Such conditions are very suitable for the proliferation and propagation of summer asphodel of Mediterranean belt rangelands.

Poisoning due to summer asphodel is not commonly found because the animals grazing in the rangelands are quite familiar with this weed plant, but it suppresses the growth of those plants having fodder value because of its high level of competitiveness. As a result of this, the quality of grass decreases and less animal products have been obtained.

Importance as Medicinal Plant

Summer asphodel has an important place in medical sciences and biopesticide sector due to containing c-glycoside; especially steroid saponins like secondary metabolized products. This weed plant stands out with its antioxidant and diuretic chemical structure. In our country, summer asphodel has a great potential in biopesticide manufacturing and medical sciences because it is very common into the rangelands vegetation coverings of Mediterranean belt. In this way, there would be a huge contribution by the widespread usage of this weed plant both in the prevention and treatment of related diseases of human being as well as for the suppression of this invasive plant and other harmful insect species found in rangelands. Many research studies have been carried out on the characteristic features of summer asphodel as a medicinal plant. One of them has been conducted on mice aimed for the treatment of gastric ulcers. In a study conducted by Gürbüz et al., (2002) on mice by using 5 different plants including summer asphodel, and the extract prepared from *A. aestivus* was shown to reduce the gastric ulcer by 94.7% and that this application did not cause lesions in the stomach of mice. In addition, in the study on medicinal plants in our country, tubers of *A. aestivus* have been used in the treatment of eczema, hemorrhoids, nephritis, burning and wounds, and in the treatment of mastitis in case of animals in the Gönen region of Balıkesir Province (Tuzlacı & Eryaşar Aymaz, 2001). Uğurlu and Seçmen (2008) also stated that the summer asphodel has been used in the treatment of wounds.

In the present scientific age, there is a great interest in finding antioxidants from natural sources to minimize oxidative damage to cells. Oxidative damage is often caused by internally produced free radicals and reactive oxygen types. These cause arteriosclerosis, cancer, diabetes, reperfusion disorder. Researchers have reported that the proper consumption of food containing antioxidants, such as herbs and

vegetables, can prevent such harmful effects of free radicals and reactive oxygen species (Peksel et al., 2013). In another study that carried out on *A. aestivus* leaves, it has been stated that the plant has a strong antioxidant activity and moderately antifungal effects (Peksel et al., 2012). In addition, it has also been emphasized that the active extracts of *A. aestivus* may be an alternative to more toxic synthetic antioxidants to food, pharmaceutical and cosmetic preparations. *A. aestivus* has antioxidant properties, and it is also reported that it is a necessary source of vitamins and essential minerals for human health (Unal & Kaplan Ince, 2017).

On the other hand, summer asphodel is also important as a bactericide as well as a fungicide in terms of human, animal and plant health. As a matter of fact, in the evaluation made with plant extracts from *A. aestivus* collected from the Aegean Region, it has been reported that most of the above-ground parts (stalk, leaf, flower), mostly the fruit, have a microbial pesticide effect (Oskay et al., 2007). In many studies, while mentioning the germicidal effect of *A. aestivus*, but the research works of İlçim et al., (1998) have emphasized that this weed plant has no effect on bacteria and fungi.

Other Areas of Usage

Although summer asphodel is a weed plant for the rangelands, but there are many different areas of its usage. This weed plant is traditionally used for food (especially, the leaves) and medical purposes in our country (Peksel et al., 2013). In the world, the tubers of summer asphodel are dried and boiled in water, and then mixed with potato pieces or wheat grain to make the 'Asphodel Bread', but in Spain and other neighbor countries, it is considered as cattle and, especially, sheep feed. In Iran, the root tubers of this weed plant are dried and converted into powder form, and then mixed with cold water to make a strong adhesive (Sawidis et. al., 2005). *A. aestivus* is also used in the production of bioethanol (Polycarpou, 2009).

Conclusion

Summer asphodel has an important feature in terms of human, animal and most of the plant health, while on the other side, it prevents the economic use of rangeland areas for animal grazing by having an aggressive weed and poisonous characteristics for the cattle and sheep in the pastures. As the rangelands are a great source, there is a potential to reduce the weed problem in rangelands by the usage of this weed plant to protect other hay grass and plants against different fungal, viral and bacterial diseases or protection from diseases and thus to increase its consumption for animals. In this way, both of the animal production will be contributed, and also would be taken benefit from *A. aestivus* as

natural means of protection and treatment of human, animal and plant health from certain diseases. Last but not least, the usage of summer asphodel as biopesticide (attractant and/or repellant) in the integrated pest management program against certain insect pests of agricultural crops is under consideration.

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